

Math 241 Final Exam

Fall, 2012

Name: _____

ID: _____

Signature: _____

Instructor: _____

Directions: Part A has 10 problems worth two points each. No partial credit will be awarded. Part B has 10 problems, with partial credit. Give complete and careful solutions. Show all of your work on the exam and provide as much detail as you can.

Part A

Page A1 _____

Page A2 _____

Page A3 _____

TOTAL _____

Part B

Page B1 _____

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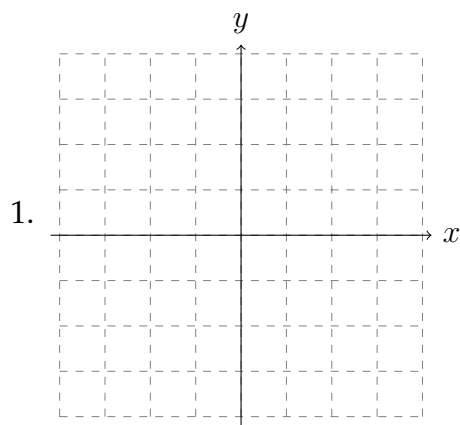
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TOTAL _____

GRAND TOTAL _____

PART A - This section contains 10 problems worth two points each. No partial credit will be awarded.



Sketch the graph of an increasing function that is defined and continuous on $(-\infty, \infty)$, and differentiable everywhere except at $x = 0$ and at $x = 2$.

2. _____

Find the limit $\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x^2 - 2x - 3}$.

3. _____

Find the limit $\lim_{x \rightarrow 1^-} \frac{|x - 1|}{x - 1}$.

4. Precisely state the “Intermediate Value Theorem for Continuous Functions”.

5. _____ Find the limit $\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 + x}}{x + 2}$.

6. _____ Find the limit $\lim_{x \rightarrow 0} \frac{\sin(7x)}{4x}$.

7. _____ What is the absolute minimum value of the function $f(x) = x^4 - 2x^2$ on the interval $[-2, 2]$?

8. _____ Find the general antiderivative of $\cos(2x) + x^2$.

9. _____ Use four rectangles of equal width and the right-endpoint values to find the Riemann sum for $f(x) = x^3 + x$ on the interval $[0, 4]$.

10. _____ Evaluate: $\int_1^8 \frac{1}{\sqrt[3]{x}} dx$

PART B - This section contains 10 problems, with partial credit. Give complete and careful solutions. Show all of your work on the exam and provide as much detail as you can

1. (8 pts.) Using only the definition of “derivative”, find $f'(x)$, where $f(x) = \sqrt{2x}$.

Answer _____

2. (6 pts.) Find the equation of the line tangent to the graph of

$$y^4 - 3xy^3 + y^2 = 6x^3 - 5x^2y$$

at the point $(1, 2)$.

Answer _____

3. (8 pts.) Water is withdrawn from a conical reservoir 20 feet in diameter and 20 feet deep (vertex down) at a constant rate of 8 cubic feet per minute. How fast is the water level falling at the instant when the depth of the water is 8 feet? [Hint: $V_{\text{cone}} = \frac{1}{3}\pi r^2 h$].

Answer _____

4. (8 pts.) Use differential approximation, or the linearization method, to estimate $\sqrt[4]{15.5}$.

Answer _____

5. (10 pts.) Given $f(x) = \frac{1-x^2}{1+x^2}$ with $f'(x) = \frac{-4x}{(1+x^2)^2}$ and $f''(x) = \frac{12x^2-4}{(1+x^2)^3}$

1 pt. _____ List any horizontal or vertical asymptotes of $f(x)$.

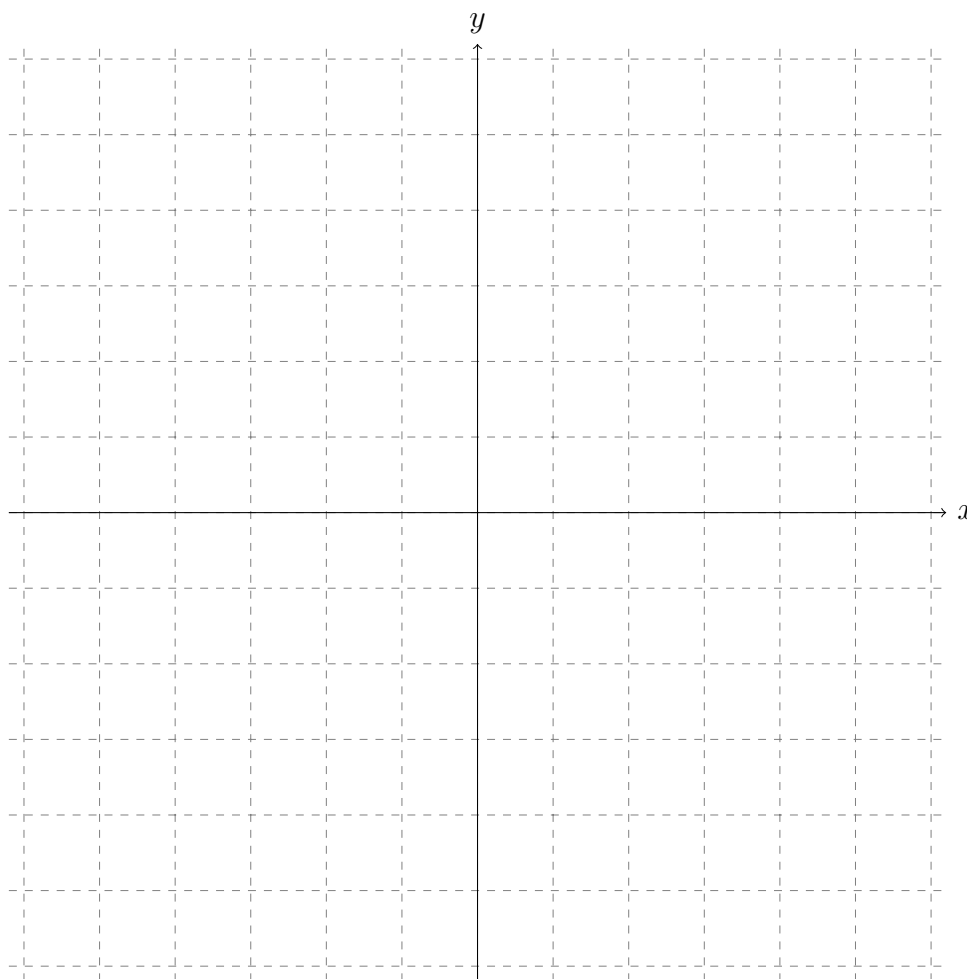
1 pt. _____ On what interval(s) is $f(x)$ increasing?

1 pt. _____ On what interval(s) is $f(x)$ concave down?

1 pt. _____ Identify any local extrema (give both coordinates).

1 pt. _____ Locate the x -coordinate of any points of inflection.

5 pts. Sketch the graph of $f(x)$, marking clearly the asymptotes, coordinates of any local extrema and points of inflection.



6. (8 pts.) A fiber board shipping crate with square base and top is constructed with double thickness on the bottom for added strength. If the volume of the crate is 96 cubic feet, find the dimensions which will minimize the needed material.

Answer _____

7. (8 pts.) If $f(x) = \int_1^x \sqrt{1 + \sin t} \, dt$, what is $f''(0)$?

Answer _____

8. (8 pts.) Evaluate the following integral: $\int_{-\pi/3}^{\pi/3} \sin x (4 + 3 \cos x) dx$.

Answer _____

9. (8 pts.) Find the area of the region bounded by the parabolas, $y = x^2 - 1$ and $y = 1 - x^2$.

Answer _____

10. (8 pts.) Consider the region bounded by $y = x^2 + 1$, and $y = 2$. Find the volume formed when this region is rotated about the x -axis.

Answer _____