Math 241

Fall 2015

Final Exam

Name: _____

Section Number: _____

Instructor: _____

Question	Points	Score
1	9	
2	9	
3	6	
4	24	
5	10	
6	8	
7	12	
8	8	
9	6	
10	16	
11	9	
12	20	
13	4	
14	9	
Total:	150	

Read all of the following information before starting the exam.

- Electronic devices (calculators, cell phones, computers), books, and notes are not allowed.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work and make clear what your final solution is (e.g. by drawing a box around it).
- This test has 14 pages total including this cover sheet and is worth 150 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. Compute the following limits. If the limit does not exist, state so. If the limit is either positive or negative infinity, say which.

   (a) (3 points)

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 2x}$$

   (b) (3 points)

$$\lim_{x \to \infty} \frac{7 - \sqrt{x}}{7 + \sqrt{x}}$$

   (c) (3 points)

$$\lim_{x \to 0} x \sin \frac{1}{x}$$

2. Consider the limit

$$\lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}$$

(a) (5 points) Find the limit.

(b) (4 points) This limit represents the derivative of some function $f(x)$ at some number a. Find $f(x)$ and a

$f(x) =$

$a =$

3. (6 points) Find all numbers a such that the function

$$f(x) = \begin{cases} 2x^3 & \text{if } x < a \\ x^4 + x^2 & \text{if } x \geq a \end{cases}$$

is continuous. *Please circle your answer.*

4. Differentiate the following functions. Do not simplify your answers.

(a) (6 points)

$$F(x) = \sqrt{x}\,(x^5 + 3)$$

(b) (6 points)

$$g(x) = 7(\sin(x^5))^2$$

(c) (6 points)

$$f(x) = \frac{\tan(\pi^3 x)}{x^2 + 1}$$

(d) (6 points)

$$g(x) = \int_x^{x^2} \frac{1}{t^3 + 1}\, dt$$

5. (10 points) Find an equation of the tangent line to the graph of the function $f(x) = x\cos x$ at $x = \pi/2$.

6. (8 points) Consider the curve

$$x^3 + \sin y = x^3 y.$$

Find the x-coordinate of a point where the curve has a horizontal tangent line.

7. (12 points) Find maximum and minimum values of the function

$$f(x) = \frac{x}{(2x+1)^2}$$

on the interval $0 \leq x \leq 2$.

8. (8 points) Two straight streets meet at right angles. Person A is walking on one of the streets towards the East at 2 miles per hour, and person B is walking on the other street towards the South at 1 mile per hour; both are walking towards the intersection. When A is 3 miles from the intersection and B is 4 miles from the intersection, at what speed does the distance between A and B shrink?

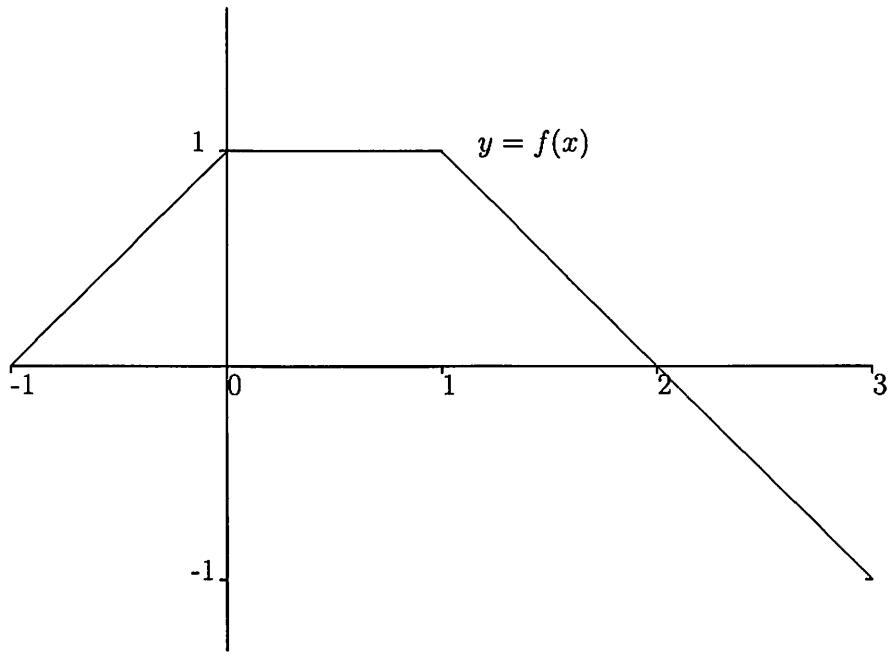
9. (6 points) Use linear approximation to estimate the number $(.95)^{10}$.

10. The graph of a function f is given below.

Set

$$g(x) = \int_0^x f(t)\, dt.$$

Note that the graph of the function f is given directly below. All the questions below are about the function g, NOT f.



(a) (2 points) Find $g(0)$.

(b) (2 points) Find $g(3)$.

(c) (2 points) Is the function g differentiable on $(-1, 3)$?
Please choose and circle the correct option:

yes *no*

(d) (2 points) Indicate all critical numbers for the function g on the interval $(-1, 3)$.

(e) (2 points) For which value of x does the function $g(x)$ take its maximum value on the interval $[-1, 3]$?

(f) (2 points) What is the maximum value of the function $g(x)$ on the interval $[-1, 3]$?

(g) (2 points) What is the minimum value of the function $g(x)$ on the interval $[-1, 3]$?

(h) (2 points) Indicate the intervals where the function g is concave up.

11. (9 points) Find the maximum area of a rectangle which has two vertices on the x-axis, and another two vertices on the parabola $y = 27 - x^2$ above x-axis.

12. Evaluate the integrals

(a) (6 points)

$$\int_0^{2\pi} \frac{5+\cos x}{3}\, dx$$

(b) (6 points)

$$\int \frac{x-5}{(x^2-10x)^2}\, dx$$

(c) (8 points)

$$\int_0^{\pi/3} 160(\cos x)^4 \sin x \, dx$$

13. (4 points) Set up (do not compute) an integral for the area of the region bounded by the curves

$$y = \sin x, \quad y = \sin x \cos x$$

between $x = 0$ and $x = \pi$.

14. (9 points) The region inside the first quadrant ($x \geq 0,\ y \geq 0$) bounded by

$$y = 4x^4,\ x = 0,\ y = 4$$

is rotated around the y-axis. Find the volume.