Math 241, Fall 2016, Final Exam

Name and section number:

Question	Points	Score
1	6	
2	6	
3	8	
4	12	
5	6	
6	8	
7	8	
8	18	
9	10	
10	9	
11	15	
12	6	
13	8	
Total:	120	

- You may not use notes or electronic devices on the test.
- Please ask if anything seems confusing or ambiguous.
- Show all your work (unless for a multiple choice question).
- You do **not** need to simplify your answers.
- Good luck!

1. Choose the option that best describes the limit in each case.

   (a) (2 points) $\lim_{x \to 2} \dfrac{x^3 - 2}{x^2 + x + 1}$.

   (a) 4/5, (b) 5/6, (c) 6/7, (d) does not exist.

   (b) (2 points) $\lim_{x \to \infty} \dfrac{4 - 7x^2}{(x+5)^2}$.

   (a) 4, (b) -7, (c) 5, (d) does not exist.

   (c) (2 points) $\lim_{x \to 3^+} \dfrac{x^2 - 2x + 1}{x - 3}$.

   (a) 0, (b) 1, (c) ∞, (d) $-\infty$.

2. (6 points) Explain why the function $f(x) = x^4 - 4x^3 + 1$ has a zero in the interval $[0, 1]$. Be clear about which theorem, or theorems, you use in your explanation.

3. (8 points) **Using the definition of the derivative as a limit**, compute $f'(4)$ if $f(x) = \sqrt{x}$.

(Warning: you will get no credit if you use the rules of differentiation).

4. Differentiate the following functions. You do not need to simplify your answers.

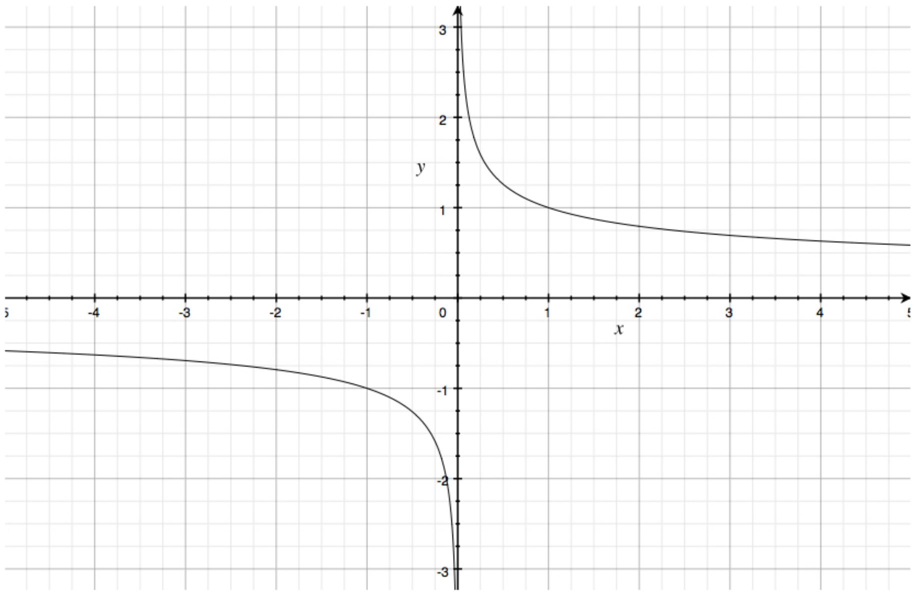
(a) (4 points) $f(x) = \dfrac{1 + x^{1/3}}{1 + x}$.

(b) (4 points) $f(x) = (x^3 + 1)^3$.

(c) (4 points) $f(x) = x \tan(x)$.

5. (6 points) Use either linearization / linear approximation, or a differential together with the fact that $\sqrt{9} = 3$ to find an approximation to $\sqrt{10}$.

6. The graph below is of the equation $y^3 x = 1$.



(a) (6 points) Find an equation for the tangent line to this graph at the point $(-1, -1)$.

(b) (2 points) Sketch the tangent line computed above on the graph.

7. (8 points) A container is in the shape of a downward pointing cone. The container is 2 meters high, and has a radius at the top end of 2 meters. Water drips out at a rate of 1 cubic meter every hour. How fast is the height of water in the container changing at the point when the height is 1 meter?

   Hint: the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$*, where* r *is the radius, and* h *is the height.*

8. Consider the function $f(x) = x^3 - 2x^2$.

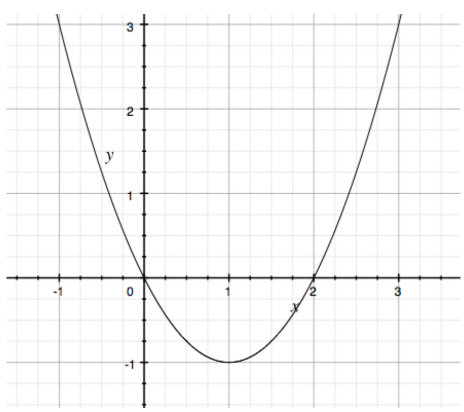
   (a) (4 points) Find all critical points of f (just give the x value(s)), and classify them as local minima, local maxima, or neither. Justify your answer for full credit.

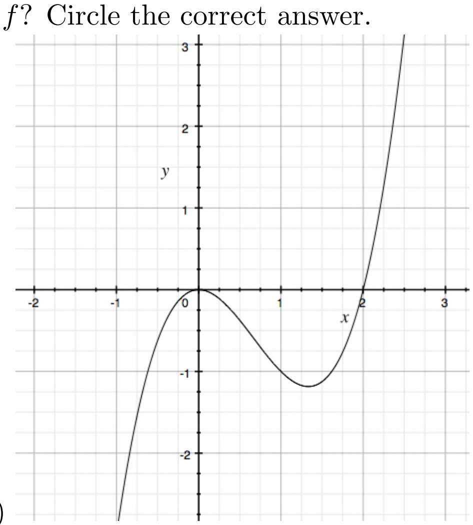
   (b) (4 points) Find all inflection points of f (just give the x value(s)). Justify your answer for full credit.

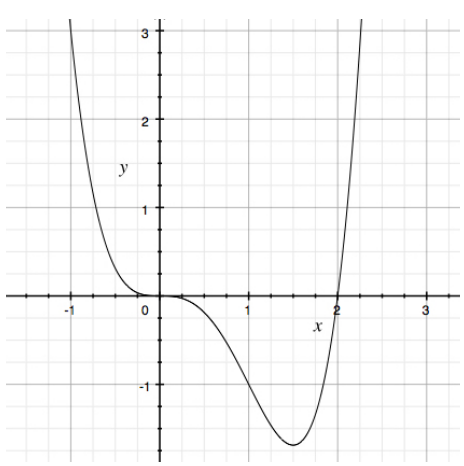
   (c) (4 points) On what intervals is f concave up, and on which is it concave down?

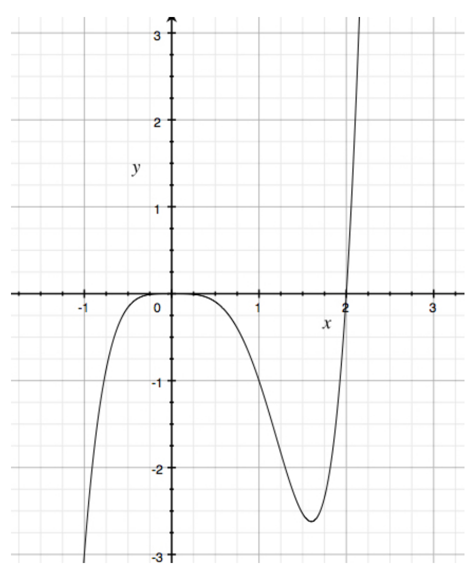
(d) (4 points) Find the absolute minimum and maximum of f on the interval $[0, 2]$, and where they occur.

(e) (2 points) Which of the following is the graph of f? Circle the correct answer.

(A) 

(B) 

(C) 

(D) 

9. (10 points) A rectangular box with a base and sides, but no top, is to be made with a volume of 1 cubic foot. The base of the box is to be square. Find the dimensions of the box which give rise to the minimum surface area.

10. Let $f(x) = 2x + 4$.

(a) (6 points) Compute a Riemann sum for this function to estimate the integral $\int_1^3 f(x)dx$. Use four equal length intervals, and use the left endpoint of each interval to give the height of the rectangles that contribute to the Riemann sum.

(b) (3 points) What is the difference between your estimate from part (a) and the actual value of $\int_1^3 f(x)dx$?

11. Compute each of the following integrals.

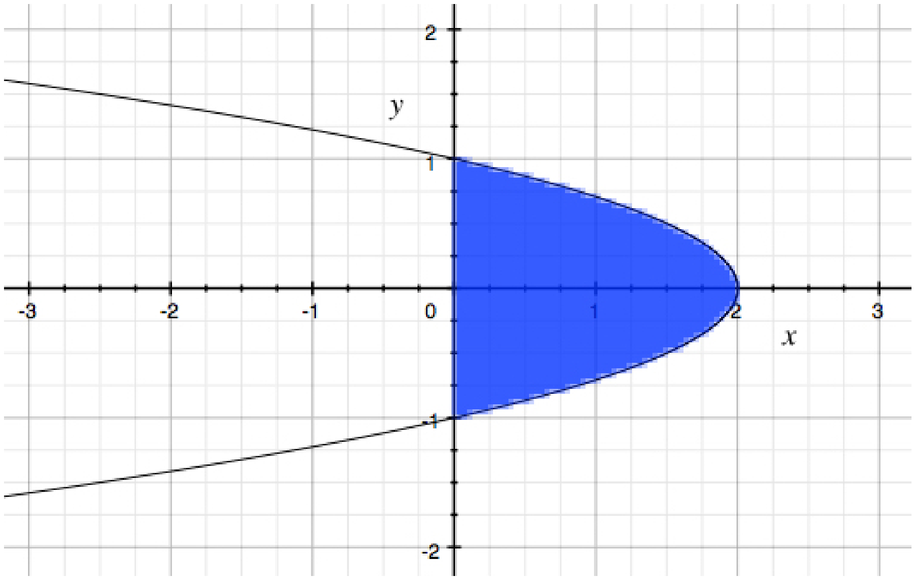
(a) (5 points) $\displaystyle\int_0^3 (1+x^2)dx.$

(b) (5 points) $\displaystyle\int_0^1 x(1+x^2)^3dx.$

(c) (5 points) $\displaystyle\int (\sqrt{x^3}-\cos(2x))dx.$

12. (6 points) Find the area of the bounded region between the graphs $y = x^3$ and $y = x^2$.

13. (8 points) The following picture shows the region between the graph of $x = 2 - 2y^2$ and the y axis.



Find the volume of the shape obtained by rotating this region about the y axis. Use any method you like.