Math 241, Fall 2018, Final Exam

Name and section number:

Instructor name:

Question	Points	Score
1	20	
2	6	
3	4	
4	20	
5	6	
6	6	
7	10	
8	10	
9	12	
10	10	
11	18	
12	8	
13	10	
14	10	
Total:	150	

- You may not use notes or electronic devices on the test.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work.
- You do **not** need to simplify your answers.
- Good luck!

1. Calculate the following limits. **Do not** use L'Hospital's rule. If the limit is positive or negative infinity, say which.

   (a) (5 points) $\lim_{x\to 3} \dfrac{x^2 - 4x + 3}{x^2 - 9}$.

   (b) (5 points) $\lim_{x\to 2^+} \dfrac{(x+1)^2}{2 - x}$.

   (c) (5 points) $\lim_{x\to 0} \dfrac{\sin 3x}{2x(x-3)}$.

   (d) (5 points) $\lim_{x\to \infty} \dfrac{x^2 + 1}{2x^2 + \sin x}$.

2. (6 points) **Using the definition of the derivative as a limit**, compute $f'(2)$ if $f(x) = \sqrt{2x+1}$. (Warning: you will get no credit if you use the rules of differentiation).

3. (4 points) Let $f(x) = \begin{cases} Ax & x \leq -1 \\ x^2 - 3Ax + 3 & x > -1 \end{cases}$.

   For which values of A is the function f continuous?

4. Differentiate the following functions. You do not need to simplify your answers.

(a) (5 points) $f(x) = \dfrac{x^3}{2x^2 - 5}$

(b) (5 points) $g(x) = 3x\sin(x^2)$

(c) (5 points) $h(x) = (\sqrt{x} - \dfrac{1}{x^4} + \pi^3)^5$

(d) (5 points) $R(x) = \displaystyle\int_1^{2x} (t + t^4)^3\, dt$

5. (a) (5 points) Use linear approximation and the fact that $\sqrt{4} = 2$ to find an approximation to $\sqrt{3.99}$.

(b) (1 point) Is the exact value for $\sqrt{3.99}$ more or less than the number you calculated in the previous part?

6. (6 points) Find an equation for the tangent line to the graph of $x^3 - 3x^2y + 2xy^2 = 0$ at the point $(1, 1)$.

7. (10 points) Superman is chasing a villain who is driving along a straight highway in a car. Superman flies at a speed of 200 feet per second, and at a constant height of 30 feet. The villain is driving at a speed of 100 feet per second. What is the rate of change of the distance between Superman and the villain when Superman is directly above a point that is 40 feet behind the villain's car?

8. (10 points) A landscape artist plans to create a rectangular garden whose area is $10\,m^2$. She plans to enclose three sides of the rectangle using trees that cost \$25 per meter, and to use fencing which costs \$20 per meter on the fourth side. Find the dimensions of the garden that will minimize her cost.

9. Let $f(x) = \dfrac{1}{x^2 - 1}$. You may use that $f' = \frac{-2x}{(x^2-1)^2}$ and $f'' = \frac{6x^2+2}{(x^2-1)^3}$.

(a) (2 points) Find the vertical asymptotes of the graph of f.

(b) (2 points) Find the horizontal asymptotes of the graph.

(c) (2 points) Find the intervals where f is increasing.

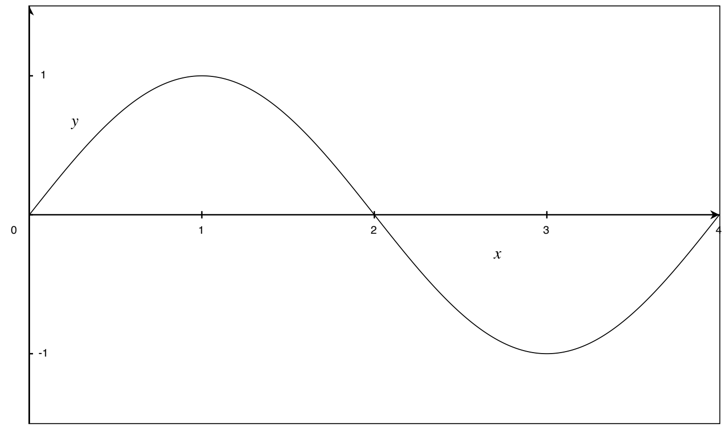
Recall that $f' = \frac{-2x}{(x^2-1)^2}$ and $f'' = \frac{6x^2+2}{(x^2-1)^3}$

(d) (2 points) Find the intervals where f is concave up.

(e) (2 points) Find the maximal value of f in the interval $[4, 6]$

(f) (2 points) Sketch of the graph of $y = f(x)$.

10. Below is the graph of the **derivative** of the function f in the interval $0 \le x \le 4$.



(a) (3 points) Find the intervals in which f is increasing.

(b) (3 points) Find the intervals in which f is concave down.

(c) (3 points) At which x between 0 and 4 does f attain its maximal value? Explain your answer.

(d) (1 point) Is it possible for the equation $f(x) = 0$ to have 3 solutions in the interval $1 \le x \le 3$?

11. Compute each of the following.

(a) (6 points) $\int (\sqrt{x} - x^{\frac{3}{2}} - \frac{4}{x^2})\, dx$

(b) (6 points) $\int_0^{\pi} (\sin x)(\cos x + 2)^3\, dx$

(c) (6 points) $\int \frac{x}{\sqrt{x^2+1}}\, dx$

12. A ball is thrown upwards from a height of 20 meters, at a speed of $15m/s$. The gravity of the earth causes the ball to accelerate **downwards** at a rate of $10m/s^2$.

 (a) (4 points) Write a function f that describes the height of the ball at time t.

 (b) (2 points) When will the ball reach its highest point?

 (c) (2 points) When will the ball hit the ground?

13. (a) (2 points) Sketch the region in the plane bounded by the lines $x = 0$, $x = 4$, $y = x$, and $y = 6 - x^2$.

(b) (8 points) Calculate the area of the region you sketched in the previous part.

14. Let R be the region bounded by the graphs $y = x^2$ and $y = 9x$.

(a) (5 points) The region R is rotated about the y-axis. Set up, but **do not evaluate** an integral describing the volume of the resulting shape. You may use any method you like.

(b) (5 points) The region R is rotated about the x-axis. Set up, but **do not evaluate** an integral describing the volume of the resulting shape. You may use any method you like.