MATH 241 COMMON FINAL EXAM, FALL 2019

You have 120 minutes.

No books, no notes, no electronic devices.

YOU MUST SHOW ALL WORK. NO NEED TO SIMPLIFY ANSWERS.

Name ______________________________

Instructor Name ______________________________

Section Number ______________________________

Grade table (for instructor's use only)

1. (16pts) __________
2. (4pts) __________
3. (8pts) __________
4. (20pts) __________
5. (5pts) __________
6. (8pts) __________
7. (6pts) __________
8. (8pts) __________
9. (10pts) __________
10. (18pts) __________
11. (15pts) __________
12. (8pts) __________
13. (10pts) __________
14. (14pts) __________

Total Score__________(/150 points)

1. Calculate the following limits. **Do not** use L'Hospital's rule. If the limit is infinite, specify whether it is ∞ or $-\infty$.

   (a) (4pts) $\lim_{x\to 5} \dfrac{x^2 - 6x + 5}{x - 5}$

   (b) (4pts) $\lim_{x\to 0} \dfrac{\sin x}{x^2 + 2x}$

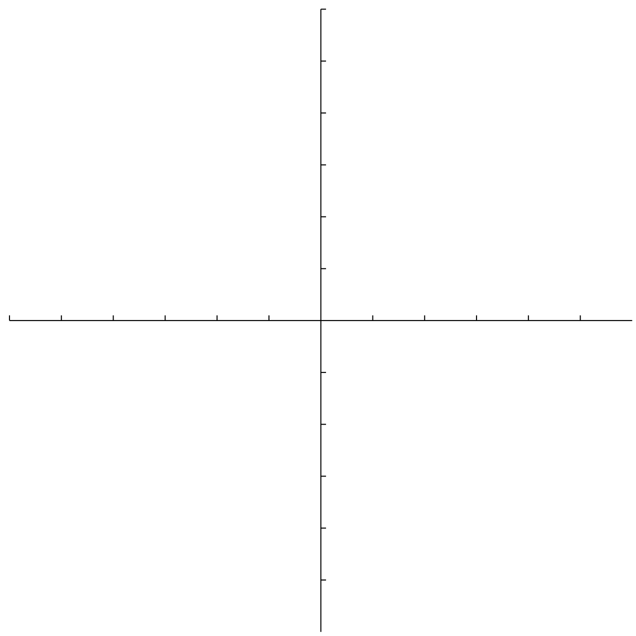
   (c) (4pts) $\lim_{x\to 2^-} \dfrac{x^2 + x - 6}{|x - 2|}$

   (d) (4pts) $\lim_{x\to \infty} \dfrac{\sqrt{2x^2 + 1}}{x}$

2. Consider the function f defined by

$$f(x) = \begin{cases} 1+x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x < 1 \\ 2 & \text{if } x = 1 \\ 2-x & \text{if } x > 1 \end{cases}$$

(a) (2pts) Sketch the graph of f.



(b) (1pt) Find the values a such that $\lim_{x \to a} f(x)$ does not exist. No justification needed.

(c) (1pt) Find the values a such that $f(x)$ is discontinuous at $x = a$. No justification needed.

3. Consider the function $f(x) = \sqrt{2x+1}$.

(a) (6pts) Using the definition of the derivative as a limit, compute $f'(0)$.
(Warning: you will not get credit if you use the rules of differentiation.)

(b) (2pts) Find the equation of the tangent line to the curve $y = f(x)$ at the point $(0, 1)$.

4. In each of the following, calculate the derivative $\frac{dy}{dx}$. You do not need to simplify your answers.

(a) (5pts) $y = \dfrac{x^2+1}{2x^2+5}$

(b) (5pts) $y = x^2 \sin\sqrt{x}$

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(c) (5pts) $y = \left(\sqrt{x} + \dfrac{1}{x} + 2\right)^5$

(d) (5pts) $y = \displaystyle\int_0^{3x} \cos^2(t)\,dt$

5. (5pts) In the following, calculate $\frac{dy}{dx}$ using implicit differentiation.

$$y \cos x = x^2 + y^2$$

6. Consider the equation $1 + x = x^3$.

   (a) (4pts) Explain why the equation has a solution in the interval $[1, 2]$. State the theorem(s) you use in your explanation.

   (b) (4pts) Explain why the equation cannot have more than one solution in the interval $[1, 2]$. State the theorem(s) you use in your explanation.

7. (6pts) Use linear approximation to estimate the number $(1.999)^4$.

8. (8pts) A balloon is rising vertically at a constant speed of 5 ft/s. A woman is driving a car along a straight road at a speed of 10 ft/s. When the woman passes directly under the balloon, it is 10 ft above her. How fast is the distance between the car and the balloon increasing 4 seconds later?

9. (10pts) A poster is to have an area of 150 cm^2 with 1 cm margins at the bottom and sides, and a 2 cm margin at the top. What dimensions (including the margins) will give the largest **printed** area?

10. Let $f(x) = \dfrac{1}{1+x^2}$. Given that $f'(x) = -\dfrac{2x}{(1+x^2)^2}$, $f''(x) = \dfrac{6x^2-2}{(1+x^2)^3}$,

(a) (2pts) find the domain of f, and find the intercepts with the x and y-axes, if there are any.

(b) (2pts) find the vertical and horizontal asymptotes of f, if there are any.

(c) (4pts) find the intervals on which f is increasing, and the intervals on which f is decreasing.

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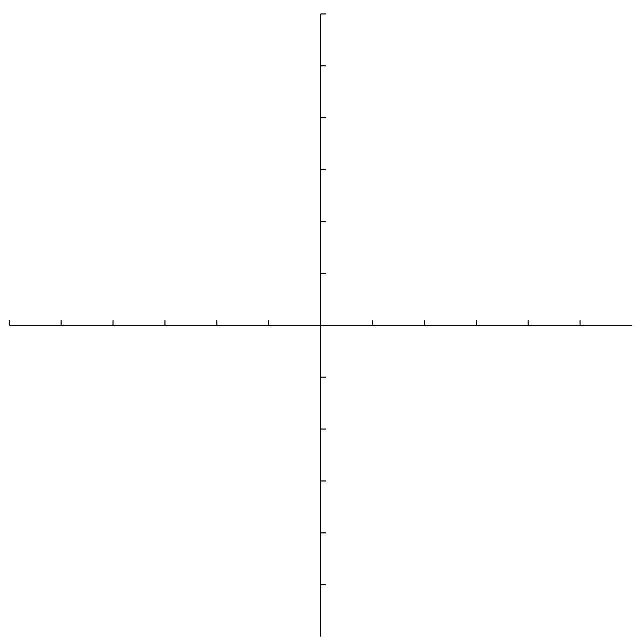
(d) (4pts) find the local minimum values and the local maximum values, if there are any.

(e) (4pts) find the intervals on which f is concave up, and the intervals on which f is concave down. Identify all inflection points, if there are any.

(f) (2pts) Sketch the graph of f.



11. Evaluate the following integrals.

(a) (5pts) $\displaystyle\int_0^{\frac{\pi}{2}} \cos x \, \sin(\sin x)\, dx$

(b) (5pts) $\displaystyle\int \frac{x+2}{\sqrt{x^2+4x}} dx$

(c) (5pts) $\displaystyle\int \frac{\sqrt{x} - 2x^2\sqrt{x} + x^4}{x}\, dx$

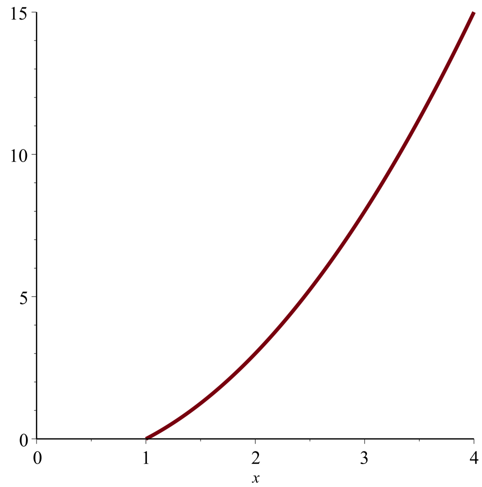
12. The height (in meters) of a projectile shot vertically upward from a point 2 m above ground level with an initial velocity of 2 m/s is $h(t) = 2 + 2t - 4.9t^2$ after t seconds.

(a) (4pts) Find the velocity of the projectile after 0.1 seconds.

(b) (2pts) What is the maximum height of the projectile?

(c) (2pts) What is the acceleration of the projectile after t seconds?

13. Consider the parabola $y = x^2 - 1$ between $x = 1$ and $x = 4$, pictured below.

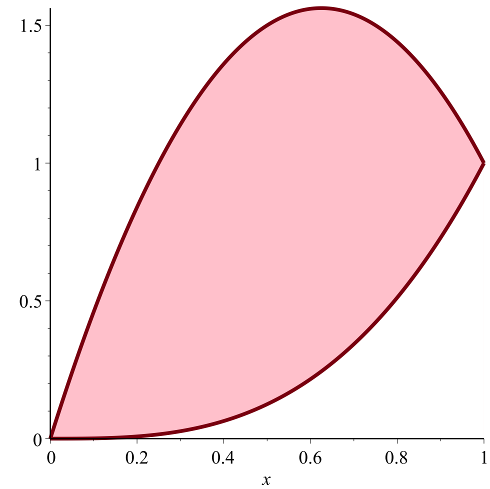


(a) (6pts) Estimate the area under the parabola and above the x-axis between $x = 1$ and $x = 4$ with a Riemann sum, using three subintervals of equal width and right endpoints.

(b) (2pts) Sketch the rectangles that you used in part (a) on the provided graph.

(c) (2pts) Is your answer in (a) larger or smaller than $\int_1^4 (x^2 - 1)\, dx$? Explain.

14. Consider the region R in the first quadrant bounded by the curves $y = x^3$ and $y = 5x - 4x^2$, pictured below. The two curves intersect at the points $(0, 0)$ and $(1, 1)$.



(a) (6pts) Find the area of the region R.

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(b) (4pts) Set up **but do not evaluate** an integral for the volume of the solid obtained by rotating R about the x-axis.

(c) (4pts) Set up **but do not evaluate** an integral for the volume of the solid obtained by rotating R about the y-axis.