Math 241 Final, Fall 2023

Name: ____________________________________

Draw a circle around your section number.

	Instructor	TA	Recitation
1	Sarah Post	Christin Sum	F 10:30-11:20a
2	Sarah Post	Christin Sum	F 12:30-1:20p
3	Hugh Chou	Axl Daguio	R 8:30-9:20a
4	Hugh Chou	Axl Daguio	R 10:30-11:20a
5	Lyon Lanerolle	Kawika O'Connor	F 9:30-10:20a
6	Lyon Lanerolle	Kawika O'Connor	F 12:30-1:20p
7	Drew Polakowski	Drew Polakowski	R 10:30-11:20a
8	Pavel Guerzhoy	Mona Aschenbrenner	T 10:30-11:20a
9	Pavel Guerzhoy	Mona Aschenbrenner	T 1:30-2:20p
10	Nicolas Antin	Alan Tong	F 10:30-11:20a
11	Nicolas Antin	Alan Tong	F 1:30-2:20p
12	Taylor Klotz	Aleksander Fedorynski	F 9:30-10:20a
13	Taylor Klotz	Aleksander Fedorynski	F 1:30-2:20p
14	Julian Hachmeister	Kawika O'Connor	R 9:30-10:20a

Question	Points	Score
1	16	
2	6	
3	6	
4	16	
5	8	
6	5	
7	8	
8	10	
9	20	
10	10	
11	15	
12	8	
13	8	
14	14	
Total:	150	

- You may not use notes or calculators on the test.
- You may not use electronic devices or access the internet.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work and make clear what your final solution is (e.g. by drawing a box around it).
- Organize your work neatly in the spaces provided and write neatly and legibly.
- Cross out any scratch work.
- You do not need to simplify your answers, unless otherwise indicated.
- You have 2 hours to complete this exam.
- Good luck!

1. Calculate the following limits. Do not use l'Hôpital's rule (don't worry if you don't know what it is). If the limit is infinite, specify whether it is $+\infty$ or $-\infty$.

   (a) (4 points) $\lim_{x \to 3} \frac{x^2 - 5x + 6}{x - 3}$

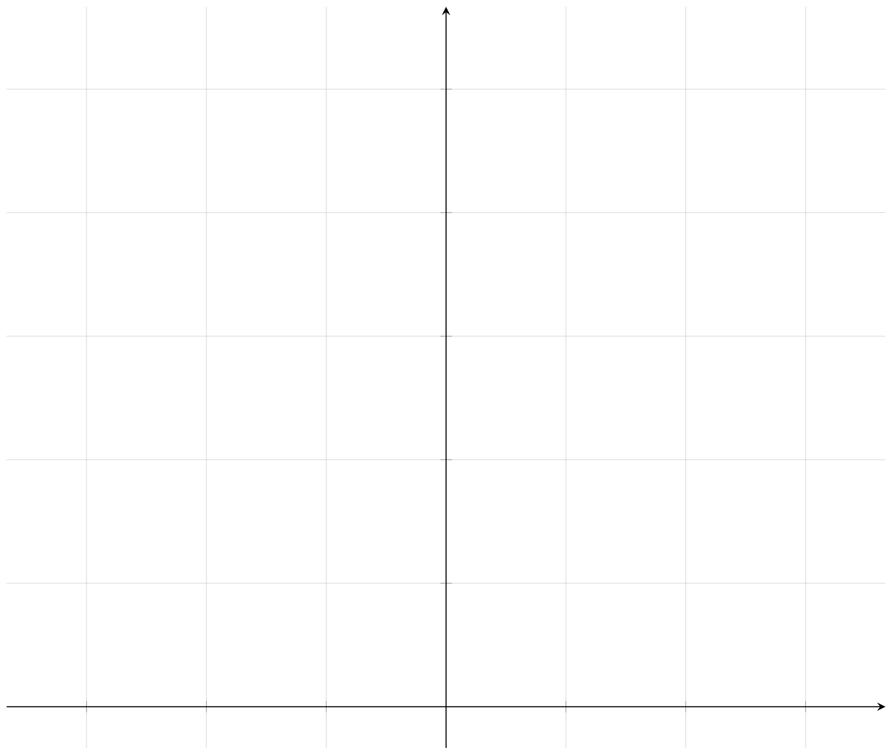
   (b) (4 points) $\lim_{x \to 0} \frac{x \cos(2x)}{\tan(2x)}$

   (c) (4 points) $\lim_{x \to 2^-} \frac{|2x - 4|}{x^2 - 4}$

   (d) (4 points) $\lim_{x \to \infty} \frac{3x^4 - 13x^3 + 31x - 4}{9x^4 + 2x^2 - 5}$

2. Consider the function $f(x) = \begin{cases} x^2, & x < 0 \\ x, & 0 \leq x \leq 1 \\ 2, & x > 1 \end{cases}$.

(a) (2 points) Sketch the graph of f.



(b) (2 points) Find all values of x at which f is discontinuous. No justification needed.

(c) (2 points) Find all values of x at which f is continuous but not differentiable. No justification needed.

3. (6 points) Let $f(x) = \dfrac{1}{x^2+1}$. Using the definition of the derivative as a limit, calculate $f'(0)$. (**Warning:** Make sure to write $f'(0)$ as a limit. You will not get credit if you use the rules of differentiation.)

4. Differentiate the following functions. You do not need to simplify your answers.

(a) (4 points) $f(x) = \dfrac{1+2x}{3x^2+2}$

(b) (4 points) $f(x) = \sin x + x \tan x$

(c) (4 points) $f(x) = \left((3+x^7)^{\frac{3}{5}} - 7x\right)^{23}$

(d) (4 points) $f(x) = \displaystyle\int_2^{\sqrt{x}} \frac{2t^2 dt}{1+\cos^2(\sqrt{t})}$

5. (8 points) Find an equation of the tangent line to the curve $x^2y^3 + x^3y^2 = 2y$ at $(1, 1)$.

6. (5 points) Use a linear approximation and the fact that $\sqrt[4]{16} = 2$ to estimate the value of $\sqrt[4]{17}$. You do not need to simplify your answer.

7. Consider the equation $4x^5 + 3x^3 + 17x = 15$.

   (a) (4 points) Explain why the equation has a solution in the interval $[0, 1]$. You may use the Intermediate Value Theorem.

   (b) (4 points) Explain why the equation cannot have more than one solution in the interval $[0, 1]$. You may use Rolle's Theorem or the Mean Value Theorem.

8. (10 points) A balloon is rising vertically over a point A on a level field at the rate of 15 ft/sec. Another point, B, on the field is 30 ft from point A. When the balloon is 40 ft above point A, at what rate is the distance between the balloon and point B changing?

9. Given $f(x) = \dfrac{x^2 - x + 1}{(x-1)^2}$, $f'(x) = -\dfrac{x+1}{(x-1)^3}$, $f''(x) = \dfrac{2x+4}{(x-1)^4}$, do the following:

(a) (2 points) Find the domain of f.

(b) (2 points) Find equations of all vertical asymptotes of f, if there are any.

(c) (2 points) Find equations of all horizontal asymptotes of f, if there are any.

(d) (2 points) Find all local maximum values of f and all local minimum values of f, if there are any.

(e) (2 points) Identify all inflection points, (x, y), of f, if there are any.

The problem continues on the next page ...

$$\left(\textit{Problem (9), continued. Recall that } f(x) = \frac{x^2 - x + 1}{(x-1)^2},\ f'(x) = -\frac{x+1}{(x-1)^3},\ f''(x) = \frac{2x+4}{(x-1)^4}. \right)$$

(f) (4 points) Find intervals on which f is decreasing and the intervals on which f is increasing.

(g) (4 points) Find the intervals on which f is concave up, and the intervals on which f is concave down.

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$\left(\textit{Problem (9), continued. Recall that } f(x) = \dfrac{x^2 - x + 1}{(x-1)^2},\ f'(x) = -\dfrac{x+1}{(x-1)^3},\ f''(x) = \dfrac{2x+4}{(x-1)^4}.\right)$

(h) (2 points) Sketch the graph of the function f and mark the x-coordinates of local minima and local maxima as well as of inflection points. Note that there are no x-intercepts and the y-intercept is at $y = 1$.

10. (10 points) An open rectangular box with a square base is to be built to hold a volume of 9 ft^3 at a cost of \$2 per ft^2 for the base and \$3 per ft^2 for the sides. Find the most economical dimensions.

11. Evaluate the following integrals.

(a) (5 points) $\displaystyle\int_1^4 \frac{2-5x^2-3x}{\sqrt{x}}dx$

(b) (5 points) $\displaystyle\int 6x\cos(x^2+1)dx$

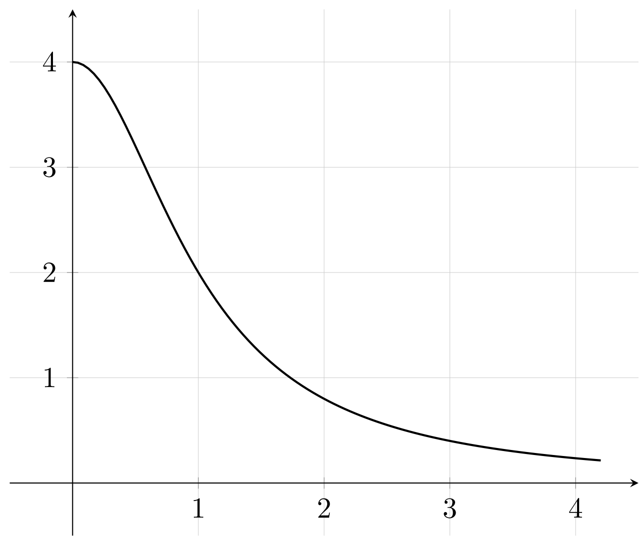
(c) (5 points) $\displaystyle\int_0^2 \frac{9x^2dx}{(x^3+2)^2}$

12. A driver applies the brakes in a car going at 80 ft/sec on a straight road. The brakes cause a constant deceleration of 10 ft/sec^2.

    (a) (4 points) How much time will it take for the car to stop?

    (b) (4 points) How far does the car move after the brakes are applied until it stops completely?

13. Consider the function $f(x) = \dfrac{4}{x^2+1}$, pictured below, on the interval $[0, 3]$.

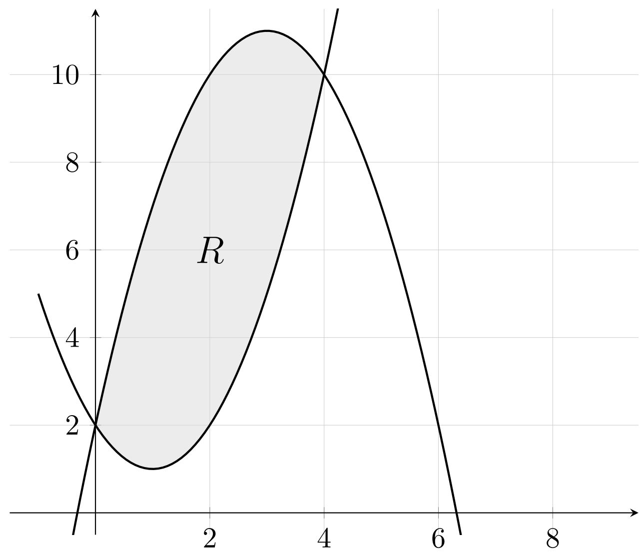


(a) (4 points) Estimate $\displaystyle\int_0^3 f(x)dx$ with a Riemann sum using 3 subintervals of equal width and left endpoints. You do not need to simplify your answer.

(b) (2 points) In the plot above, sketch the rectangles that you used in part (a).

(c) (2 points) Which is larger: your estimate from part (a) or the exact value of $\displaystyle\int_0^3 f(x)dx$.

14. Consider the region R bounded by the curves $y = 2 + 6x - x^2$ and $y = x^2 - 2x + 2$.



(a) (6 points) Find the area of the region R.

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(b) (4 points) Set up **but do not evaluate** an integral for the volume of the solid obtained by rotating R about the x-axis.

(c) (4 points) Set up **but do not evaluate** an integral for the volume of the solid obtained by rotating R about the y-axis.