**Math 241, Spring 2012** **Name:**
**Final Exam** **Instructor:**

**Instructions:** Write legibly. To earn full credit, you must show enough of your work to justify your answers. Turn off and store all of your electronic devices; this includes calculators, cell phones, tablets and music players. All work should be your own.

| **Problem** | **Worth** | **Score** |
|---|---|---|
| 1 | 20 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 20 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 20 | |
| 8 | 10 | |
| 9 | 20 | |
| 10 | 10 | |
| 11 | 10 | |
| 12 | 10 | |
| Total | 160 | |

**Extra Credit** (5 points)**.** Evaluate the following derivative: $\dfrac{d}{dx}\displaystyle\int_{x^2}^{\sin x} \sqrt{1+t^2}\, dt.$

**Problem 1** (20 points). Evaluate each of the following limits or show that they do not exist. **Show your work!**

(a) $\displaystyle\lim_{x\to -2} \frac{x^2 - 4x + 5}{x^2 - 2}$

(b) $\displaystyle\lim_{x\to 2^-} \frac{x^2 - 4}{|x + 2|}$

(c) $\displaystyle\lim_{x\to \infty} \frac{\sin(x^2)}{\sqrt{x}}$

(d) $\displaystyle\lim_{x\to 0} \frac{1 - \cos x}{x}$

**Problem 2** (10 points)**.** A particle moves in a straight line along the $s$-axis. At time $t$, its acceleration is $a(t) = -6t + 2$. Its position and velocity at time $t = 4$ are $s(4) = 1$ and $v(4) = 1$, respectively. Find the position function $s(t)$.

**Problem 3** (10 points)**.** Find an equation for the tangent line to the curve given by

$$2x^2y - 3xy^2 = 16$$

at the point $(-1, 2)$.

**Problem 4** (20 points)**.** Find the derivative of each of the following functions.
**Do not simplify!**

(a) $f(x) = \sqrt{5 + \dfrac{2}{x^6}}$

(b) $g(x) = x^7 \tan x$

(c) $h(x) = \dfrac{\cos(7 \sin x)}{8 + \sec(2x)}$

(d) $j(x) = \displaystyle\int_2^x \frac{t^5}{7 + t^8}\, dt.$

**Problem 5** (10 points)**.** Consider the equation $x^3 - 7x + 1 = 0$.

(a) Does this equation have a solution in the interval $[2, 3]$? Justify your answer.

(b) Does this equation have more than one solution in this interval?

**Problem 6** (10 points)**.** Using the **limit definition** of the derivative, find the derivative of

$$f(x) = x^2 + 3x + 1$$

at $x = 2$. **To receive credit, you must show your work. It is not acceptable to use differentiation rules.**

**Problem 7** (20 points)**.** Below you are given a function $f(x)$ and its first and second derivatives. Use this information to solve the following problems.

$$f(x) = \frac{x^2-4}{x^2+1} \qquad f'(x) = \frac{10x}{(x^2+1)^2} \qquad f''(x) = \frac{10(1-3x^2)}{(x^2+1)^3}$$

(a) Find the global maximum and minimum value of $f(x)$ on the interval $[-2, 3]$.
**Show your work!**

(b) Determine the intervals where the function is increasing and the intervals where it is decreasing.

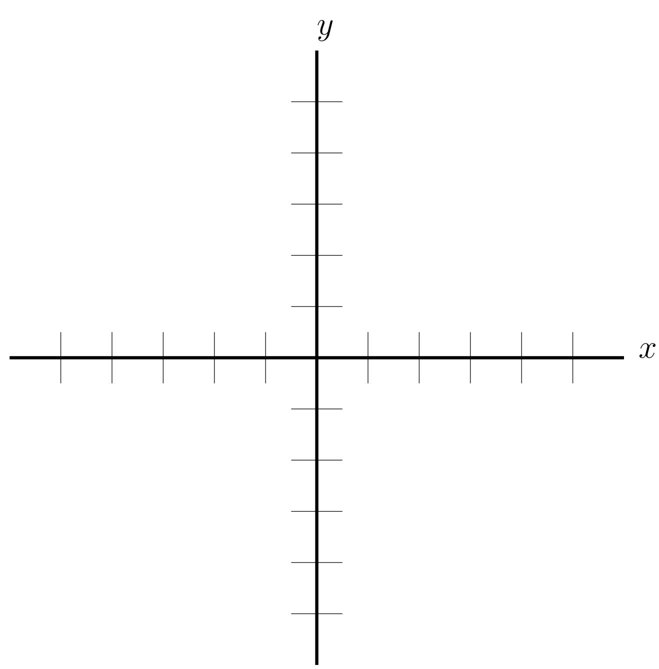
(c) Find the $x$-coordinate of each local extremum (local maximum and minimum).

(d) Determine the intervals where the function is concave up and the intervals where it is concave down.

(e) Determine the $x$-values where the inflection points occur.

(f) Determine all vertical and horizontal asymptotes.

(g) Sketch the graph of $y = f(x)$.



**Problem 8** (10 points)**.** A box with no top is constructed by cutting equal-sized squares from the corners of a 12cm $\times$ 12 cm sheet of metal and bending up the sides. What is the largest possible volume of such a box?

**Problem 9** (20 points). Evaluate the following integrals. **Show your work!**

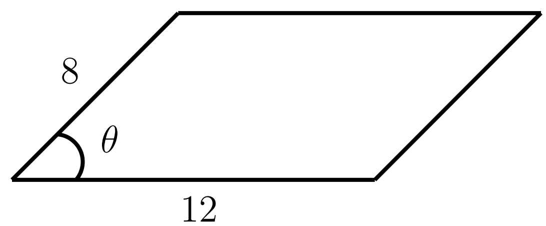
(a) $\displaystyle\int_{-1}^{1} (x^3 - x)\, dx$

(b) $\displaystyle\int \frac{dx}{\cos^2 x}$

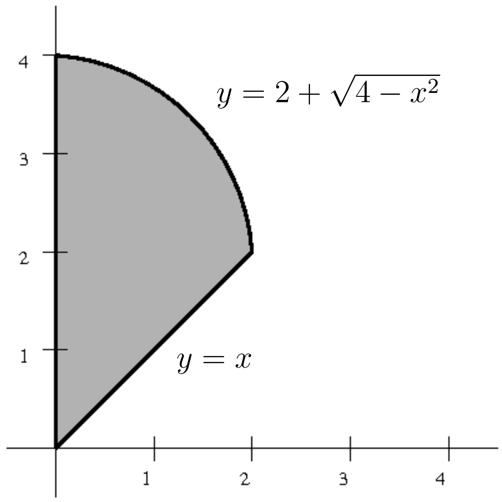
(c) $\displaystyle\int_{0}^{2} x\sqrt{9 - 2x^2}\, dx$

(d) $\displaystyle\int \frac{2x^2 - 3x}{x}\, dx$

**Problem 10** (10 points)**.** A parallelogram has fixed side lengths 8cm and 12cm. The indicated angle $\theta$ is increasing at a rate of $\pi/4$ radians per second. How fast is the area changing when $\theta = \pi/3$ radians?



**Problem 11** (10 points). Consider the shaded region of the plane pictured below. It is bounded on the left by the $y$-axis, below by the line $y = x$, and above by the graph of $y = 2 + \sqrt{4 - x^2}$.




(a) Express the area of the shaded region using one or more *unevaluated* definite integrals.

(b) Find the volume of the solid of revolution given by rotating the shaded region about the $y$-axis.

**Problem 12** (10 points)**.** Determine the values of the parameters $a$ and $b$ such that the following function $f(x)$ becomes continuous and differentiable at $x = 2$:

$$f(x) = \begin{cases} x^2 - 2x + b & \text{for } x > 2 \\ ax & \text{for } x \leq 2. \end{cases}$$