

# Math 241 Final Exam, Spring 2014

Name:

Section number:

Instructor:

Question	Points	Score
1	15	
2	15	
3	10	
4	15	
5	10	
6	10	
7	10	
8	10	
9	15	
10	10	
11	10	
12	20	
Total:	150	

**Read all of the following information before starting the exam.**

- Calculators and notes are not allowed.
- Show all work clearly. You may lose points if we cannot see how you arrived at your solution.
- Box or otherwise clearly indicate your final answers.
- This test has 10 pages total including this cover sheet and is worth 150 points. It is your responsibility to make sure that you have all of the pages!

1. (15 points) Compute the following limits or show that they do not exist:

(a)

$$\lim_{t \rightarrow 4} \left( \frac{\sqrt{t} - 3}{t - 9} \right)$$

(b)

$$\lim_{x \rightarrow 0} \frac{x}{\sin 3x}$$

(c)

$$\lim_{x \rightarrow \infty} \frac{x \sin(x^2)}{x^2 + 1}$$

2. (15 points) Find the derivative of the following functions. Do not simplify.

(a)  $f(x) = x^3\sqrt{\sin x}$

$$f'(x) =$$

(b)  $g(x) = \tan(2 + x^2)$

$$g'(x) =$$

(c)  $h(x) = \int_2^{x^3} \frac{\sin x}{x} dx$

$$h'(x) =$$

3. (10 points) Using the **definition of derivative**, **NOT differentiation rules**, find  $f'(2)$  if

$$f(x) = \frac{1}{3x}.$$

4. Evaluate the following integrals. **Show your work!**

(a) (5 points)  $\int \sin(x) \cos^3(x) dx$

(b) (5 points)  $\int_0^1 (t^2 + 1)(4t - 1) dt$

(c) (5 points)  $\int x^2 \sqrt{x+1} dx$

5. (10 points) Find the area of the region bounded by the lines  $y = x$ ,  $y = 8x$  and the curve  $y = 1/x^2$ .

6. (10 points) Find an equation for the tangent line to the curve defined by

$$x^3 + y^3 = 9xy$$

at the point  $(4, 2)$ . Show your work!

7. (10 points) Consider a point  $P = (x, y)$  that moves along the graph of the function  $y = 8/x$  with a horizontal velocity of 3 units per second. (This means that  $dx/dt = 3$ .) At what rate does the distance between  $P$  and the origin  $(0, 0)$  change as the point passes through  $(4, 2)$ ?

8. (10 points) A rectangular box with volume 6 cubic feet is to be built with a square base and no top. The material used for the bottom panel costs \$3.00 per square foot and the material for the side panels costs \$2.00 per square foot. Find the minimum cost of such a box. **Justify your answer using the methods of calculus.**

9. Let  $f(x) = x(x-3)^2 = x^3 - 6x^2 + 9x$ .

(a) (3 points) Compute the first and second derivatives:

$$f'(x) =$$

$$f''(x) =$$

(b) (3 points) Find the interval(s) where  $f$  is increasing and those where  $f$  is decreasing.

(c) (4 points) Find the local and global maxima and minima on the interval  $[-1, 3.5]$ . Give both the values and where they are attained.

(d) (3 points) Find the intervals where  $f$  is concave up and those where it is concave down.

(e) (2 points) Find the inflection points of  $f$ .

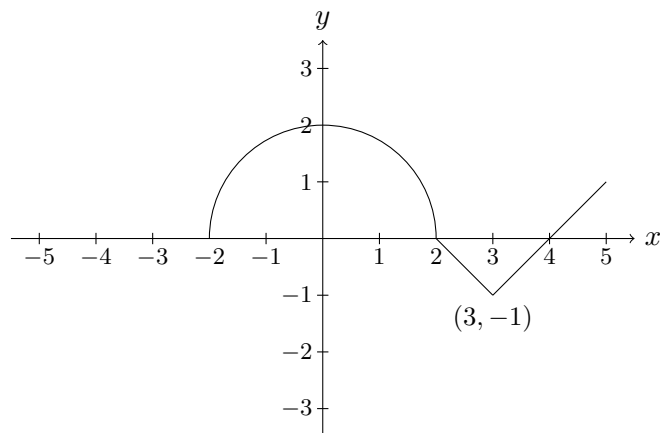


10. (10 points) Suppose  $f(x)$  and  $g(x)$  are both positive, increasing, and concave upward on an interval  $I$ . Also suppose that  $f(x)$  and  $g(x)$  are twice-differentiable; that is, suppose  $f'(x)$ ,  $f''(x)$ ,  $g'(x)$  and  $g''(x)$  all exist. Show that  $f(x)g(x)$  is also concave upward on  $I$ .

11. (10 points) Let  $R$  be the region bounded by the graph of  $f(x) = 1 - (x - 2)^2$  and the x-axis. Set up but **do not evaluate** integrals for the following:

(a) the volume of the region arrived at by rotating  $R$  about the x-axis.

(b) the volume of the region arrived at by rotating  $R$  about the y-axis.



12. (20 points)

The graph of a function  $f$  consists of a semicircle and two line segments as shown above. Let  $g$  be the function given by

$$g(x) = \int_0^x f(t) dt.$$

- (a) Find  $g(3)$ .
  
- (b) Find all values of  $x$  on the open interval  $(-2, 5)$  at which  $g$  has a relative maximum. Justify your answer.
  
- (c) Write an equation for the line tangent to the graph of  $g$  at  $x = 3$ .
  
- (d) Find the  $x$ -coordinate of each point of inflection of the graph of  $g$  on the open interval  $(-2, 5)$ . Justify your answer.