Math 241, Spring 2015, Final Exam

Name and section number:

Question	Points	Score
1	16	
2	8	
3	16	
4	10	
5	6	
6	10	
7	6	
8	13	
9	4	
10	3	
11	3	
12	8	
13	9	
14	12	
15	6	
16	10	
Total:	140	

- You may not use notes or electronic devices on the test.
- Please ask if anything seems confusing or ambiguous.
- Show all your work.
- Good luck!

1. Compute the following limits. **Do not** use L'Hospital's rule. If the limit is either positive or negative infinity say which. Simplify your answers.

   (a) (4 points) $\lim_{x\to 0^-} \dfrac{|x+1|}{2+\sin^2 x}$.

   (b) (4 points) $\lim_{t\to -2} \dfrac{t+2}{t^2-4}$

   (c) (4 points) $\lim_{s\to 0} f(s)$, where $f(s) = \begin{cases} s\left(3-\frac{5}{s}\right), & s \neq 0; \\ 0, & s = 0. \end{cases}$

   (d) (4 points) $\lim_{x\to\infty} \dfrac{2x}{x+7\cos x}$.

2. (8 points) Let r be the function $r(x) = \sqrt{2x+1}$. **Using the definition of the derivative as a limit**, show that $r'(0) = 1$.

   (Warning: you get no credit for using the rules of differentiation).

3. Differentiate the following functions. **Do not** simplify your answers.

(a) (4 points) $y = 3x^5 - 2x^3 - 7x + \pi$.

$y' =$

(b) (4 points) $z = \dfrac{2x \sin x}{5x + 3}$.

$\dfrac{dz}{dx} =$

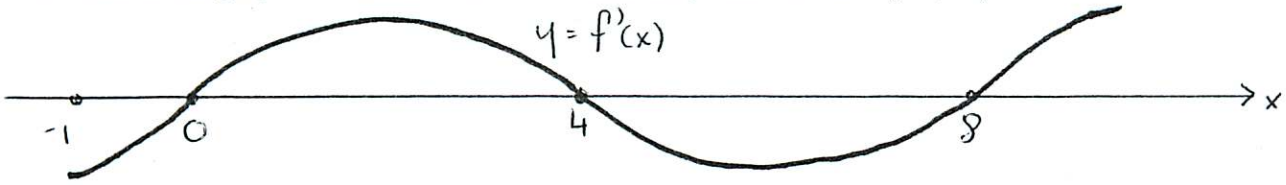
(c) (4 points) $w = \sqrt{x} + \left(3 + x^2\right)^7$.

$\dfrac{dw}{dx} =$

(d) (4 points) $R(t) = \displaystyle\int_0^{t+2} \cos(1 + x^2)\, dx$.

$R'(t) =$

4. Here is the graph of the *derivative* of a function f with domain $(-1, \infty)$.



(a) (5 points) Which is larger, $f(2)$ or $f(4)$ (1 of 4 pts)? You must explain (4 of 5 pts).

(b) (5 points) At what value(s) of x does f have a local minimum (2 pts)? Explain (3 pts).

5. Let f be the function $f(x) = (4 - 5x)^3$.

(a) (6 points) Find the equation of the tangent line to the graph $y = f(x)$ at the point $(1, -1)$.

6. A point moves along the curve $y = x^2 + 2x - 2$ in such a way that when it is at $(1, 1)$ the x-coordinate is **decreasing** at a rate of 1 unit/second. At this time what is

(a) (7 points) the rate of change of the y-coordinate of the point?

(b) (3 points) the rate of change of the distance $d = \sqrt{x^2 + y^2}$ from the point to the origin?

7. Starting with $(27)^{1/3} = 3$,

   (a) (6 points) show how differentials/linear approximation can be used to approximate $(29)^{1/3}$. Sketch a picture illustrating your computation.

8. Let $f(x) = \dfrac{(x+1)(x+3)}{x^2+3}$. Then $f'(x) = \dfrac{4(3-x^2)}{(x^2+3)^2}$ and $f''(x) = \dfrac{8x(x^2-9)}{(x^2+3)^3}$.
No need to check; you can trust me!

(a) (2 points) List all $x-$intercepts and $y-$intercepts.

(b) (3 points) List all intervals where f increases.

(c) (3 points) List all inflection points and intervals where the graph is concave down.

(d) (2 points) List any asymptotes (vertical or horizontal).

(e) (3 points) Sketch the graph $y = f(x)$.

The problems on this page are multiple choice. Circle the letter of the correct answer.

9. (4 points) Consider the integral $I = \int_0^1 \frac{1}{1+x^6}\,dx$. Then

   (a) $I < 1$.

   (b) $I = 1$.

   (c) $I > 1$.

10. (3 points) True or False? If $f(x)$ is continuous at $x = a$, then $f(x)$ is differentiable at $x = a$.

    (a) True.

    (b) False.

11. (3 points) True or False? Suppose that f and g are differentiable on $(-\infty, \infty)$ and $f'(x) = g'(x)$ for all x. If $f(0) > g(0)$, then $f(x) > g(x)$ for all x.

    (a) True.

    (b) False.

12. (8 points) Find the maximum *perimeter* of a rectangle that has its bottom two corners on the x-axis and its top two corners on the parabola $x^2 + y = 4$. (The perimeter of a rectangle is the sum of lengths of its four edges.)

13. Consider the function $f(x) = \dfrac{1}{x}$, $1 \leq x \leq 5$.

(a) (4 points) Compute a Riemann sum for this function that approximates the integral $\int_1^5 f(x)dx$. Use four equal-width intervals for your Riemann sum, and use the right endpoint of each interval to determine the height of the corresponding rectangle. You do not have to simplify your answer.

(b) (4 points) Sketch the graph $y = f(x)$, $1 \leq x \leq 5$, and the rectangles that correspond to the Riemann sum in part (a).

(c) (1 point) Does your solution to (a) overestimate or underestimate $\int_1^5 f(x)dx$?

14. Compute the following integrals.

(a) (4 points) $\int t^2(t-1)\,dt.$

(b) (4 points) $\int_0^1 (2x+1)^4 dx.$

(c) (4 points) $\int_0^1 \frac{d}{dx}\left[\frac{x(x+2)}{x^4+5}\right] dx.$

15. (6 points) Find an exact formula for $f(t)$, given that $f''(t) = 3-\cos t$, $f'(0) = 5$, and $f(0) = -2$.

16. (a) (5 points) Set-up (do not evaluate!) an integral that represents the volume of the solid generated by revolving about the y-axis the region bounded by the curves $x = 1$, $y = 5$, and $y = x^2 + 1$, $1 \leq x \leq 2$. You may use any method.

(b) (5 points) Set-up an integral (do not evaluate!) if the solid is now obtained by revolving the above region about the x-axis.