Math 241, Spring 2017, Final Exam

Name and section number:

Instructor name:

Question	Points	Score
1	16	
2	8	
3	16	
4	6	
5	6	
6	7	
7	8	
8	10	
9	12	
10	15	
11	4	
12	6	
13	8	
14	8	
Total:	130	

- You may not use notes or electronic devices on the test.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work.
- You do **not** need to simplify your answers.
- Good luck!

1. Calculate the following limits. **Do not** use L'Hospital's rule. If the limit is positive or negative infinity, say which.

   (a) (4 points) $\lim_{x\to\infty} \dfrac{7-4x-x^4}{2(x^2-2)^2}$.

   (b) (4 points) $\lim_{x\to 1^+} \dfrac{x^2-1}{(x-1)^3}$.

   (c) (4 points) $\lim_{x\to 2} \dfrac{\sqrt{x+7}-3}{x-2}$.

   (d) (4 points) $\lim_{x\to 0} \dfrac{\sin 5x}{x(x+1)}$.

2. (a) (6 points) **Using the definition of the derivative as a limit**, compute $f'(0)$ if $f(x) = \dfrac{1}{2x+1}$.
(Warning: you will get no credit if you use the rules of differentiation).

(b) (2 points) The limit $\displaystyle\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h}$ represents the derivative of some function g at some point a. What is g and what is a?

3. Differentiate the following functions. You do not need to simplify your answers.

(a) (4 points) $f(x) = \dfrac{5}{x^7} - 2x^3 + \sqrt{x} + 7\pi^2$

(b) (4 points) $g(x) = \dfrac{x^2(x^3+1)}{2-x^5}$

(c) (4 points) $h(x) = (1 + \sin(7x^2))^3$

(d) (4 points) $R(x) = \displaystyle\int_0^{3x} (1+t^3)^5 dt$

4. (6 points) Use linear approximation and the fact that $\frac{1}{100} = 0.01$ to find an approximation to $\frac{1}{102}$.

5. (6 points) Find an equation for the tangent line to the graph of $x^4 + x^2y + y^3 = 3$ at the point $(1, 1)$.

6. Consider the equation $1 + x = x^3$.

   (a) (5 points) Explain why the equation has a solution in the interval $[1, 2]$. State the theorems you use in your explanation.

   (b) (2 points) Explain why the equation can't have two solutions in the interval $[1, 2]$. State the theorems you use in your explanation.

7. (8 points) A person flies a kite at a height of 300 feet. The wind carrying the kite moves it away from the person horizontally at a speed of 25 feet per second. What is the rate of change of the length of the kite string (that is - the distance from the person to the kite), when the kite is 500 feet away from the person?

8. (10 points) A rectangular box has a base that is a square. The perimeter of the base plus the height of the box is equal to 3 feet. What is the largest possible volume for such a box, and what are its dimensions? Justify your answer.

9. Let $f(x) = 3x^5 - 5x^3$.

   (a) (2 points) find the critical points of f.

   (b) (2 points) Classify the critical points of f as local maxima, local minima, or neither.

   (c) (2 points) Find the intervals where f is increasing.

(d) (2 points) Find the maximal and minimal values of f in $[-2, 0]$.

(e) (2 points) Find the intervals where f in concave up.

(f) (2 points) Give a rough sketch of the graph of $y = f(x)$.

10. Compute each of the following.

(a) (5 points) $\int_0^{\frac{\pi}{2}} \sin(x)\cos^5(x)dx$

(b) (5 points) $\int \frac{x^2-1}{\sqrt{(x^3-3x)}}dx$

(c) (5 points) Find the function $F(x)$ given that $F'(x) = x^2 + 4x + 5$ and $F(1) = 2$.

11. Let $f(x) = x^2 - 1$. Partition the interval $[1, 4]$ into 3 equal parts.

(a) (2 points) Calculate a Riemann sum for f using the left endpoint of each interval.

(b) (2 points) Is the Riemann sum you calculated in the previous part more or less than $\int_1^4 (x^2 - 1)dx$? Explain your answer.

12. For each of the following, answer True or False. No further explanation is required.

(a) (2 points) Every differentiable function is also continuous.

(b) (2 points) The function $F(x) = \int_0^x \frac{1}{1+t^2+t^4} dt$ is increasing.

(c) (2 points) If $f'(1) = 0$ and $f''(1) = 0$ then f cannot achieve a local maximum at 1.

13. (8 points) Calculate the area bounded by the graphs of $y = x^2 - 1$ and $y = 3x + 3$.

14. Consider the region R bounded by the graphs of $y = 2x$, $y = 3 - x^2$ and $x \geq 0$.

   (a) (4 points) The region R is rotated about the y-axis. Set up, but **do not evaluate** an integral describing the volume of the resulting shape. You may use any method you like.

   (b) (4 points) The region R is rotated about the x-axis. Set up, but **do not evaluate** an integral describing the volume of the resulting shape. You may use any method you like.