MATH 241 COMMON FINAL EXAM, SPRING 2019

You have 120 minutes.

No books, no notes, no electronic devices.

YOU MUST SHOW ALL WORK. NO NEED TO SIMPLIFY ANSWERS.

Name ______________________________

Instructor Name ______________________________

Section Number ______________________________

Grade table (for instructor's use only)

1. (16pts) __________
2. (4pts) __________
3. (6pts) __________
4. (20pts) __________
5. (8pts) __________
6. (6pts) __________
7. (7pts) __________
8. (8pts) __________
9. (10pts) __________
10. (18pts) __________
11. (15pts) __________
12. (8pts) __________
13. (10pts) __________
14. (14pts) __________

Total Score__________(/150 points)

1. Calculate the following limits. **Do not** use L'Hospital's rule. If the limit is infinite, specify whether it is $+\infty$ or $-\infty$.

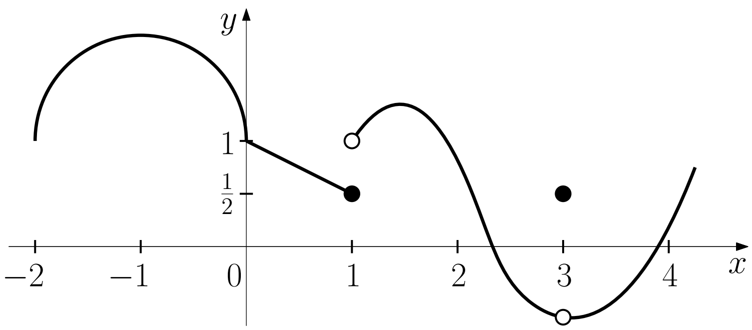
   (a) (4pts) $\lim_{x \to 2} \dfrac{x^2 - 4}{x^2 - 3x + 2}$

   (b) (4pts) $\lim_{x \to 0} \dfrac{x \cos x}{\sin x}$

   (c) (4pts) $\lim_{x \to 2^-} \dfrac{|2x - 4|}{2 - x}$

   (d) (4pts) $\lim_{x \to \infty} \dfrac{4x^3 + \sin x}{2x^3 + 3}$

2. Consider the function $f(x)$ whose graph is shown below.



(a) (1pt) Find the following limit: $\lim_{x \to 1^-} f(x)$.

(b) (1pt) Find the values a such that $\lim_{x \to a} f(x)$ does not exist.

(c) (1pt) Find the values a such that $f(x)$ is discontinuous at $x = a$.

(d) (1pt) Find the values a such that $f'(a)$ does not exist.

3. (6pts) Use the definition of the derivative to compute $f'(1)$ for $f(x) = \dfrac{4}{x+1}$.
(Warning: you will not get credit if you use the rules of differentiation.)

4. Differentiate the following functions. You do not need to simplify your answers.

(a) (5pts) $f(x) = \dfrac{x-1}{x+1}$

(b) (5pts) $f(x) = (x-1)\cos(1+x^2)$

Problem continues on the next page...

(c) (5pts) $f(x) = \left(x^2 + \dfrac{1}{\sqrt{x+1}} + 3^3\right)^{\frac{3}{2}}$

(d) (5pts) $f(x) = \displaystyle\int_0^{x^2} \frac{dt}{1+\sin^2 t}$

5. Consider the equation $x^5 + 2x - 1 = 0$.

   (a) (6pts) Use the Intermediate Value Theorem to show that the given equation has a solution in the interval $[0, 1]$.

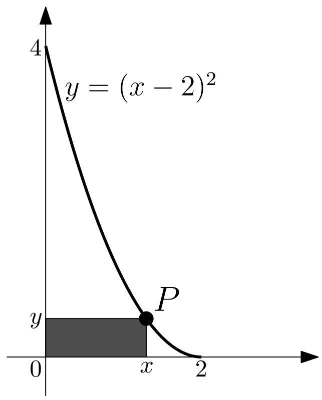
   (b) (2pts) Use Rolle's theorem or the Mean Value Theorem to show that the given equation cannot have more than one solution in the interval $[0, 1]$.

6. (6pts) Use linear approximation and the fact that $27^{-\frac{1}{3}} = \dfrac{1}{3}$ to estimate $28^{-\frac{1}{3}}$.

7. (7pts) Find an equation of the tangent line to the curve $x^2y^2 - 2x = 9 - y$ at the point $(-2, 1)$.

8. (8pts) A stone dropped in a pond sends out a circular ripple whose radius increases at a constant rate of 3 ft/sec. How rapidly is the area enclosed by the ripple increasing at the moment when the radius is equal to 30 ft?

9. (10pts) Consider the parabola $y = (x-2)^2$. Find the coordinates (x, y) of the point P lying on this parabola between $x = 0$ and $x = 2$ such that the *perimeter* of the rectangle shown below is the smallest.



10. Let $f(x) = \dfrac{x+1}{(x-1)^2}$. Then $f'(x) = -\dfrac{x+3}{(x-1)^3}$, $f''(x) = \dfrac{2(x+5)}{(x-1)^4}$.

(a) (4pts) Find vertical and horizontal asymptotes of the graph of $f(x)$, if there are any.

(b) (4pts) Find intervals on which $f(x)$ is increasing and intervals on which it is decreasing. Find critical numbers of $f(x)$, if there are any.

(c) (2pts) Find x-coordinates of local minima and local maxima of $f(x)$, if any exist.

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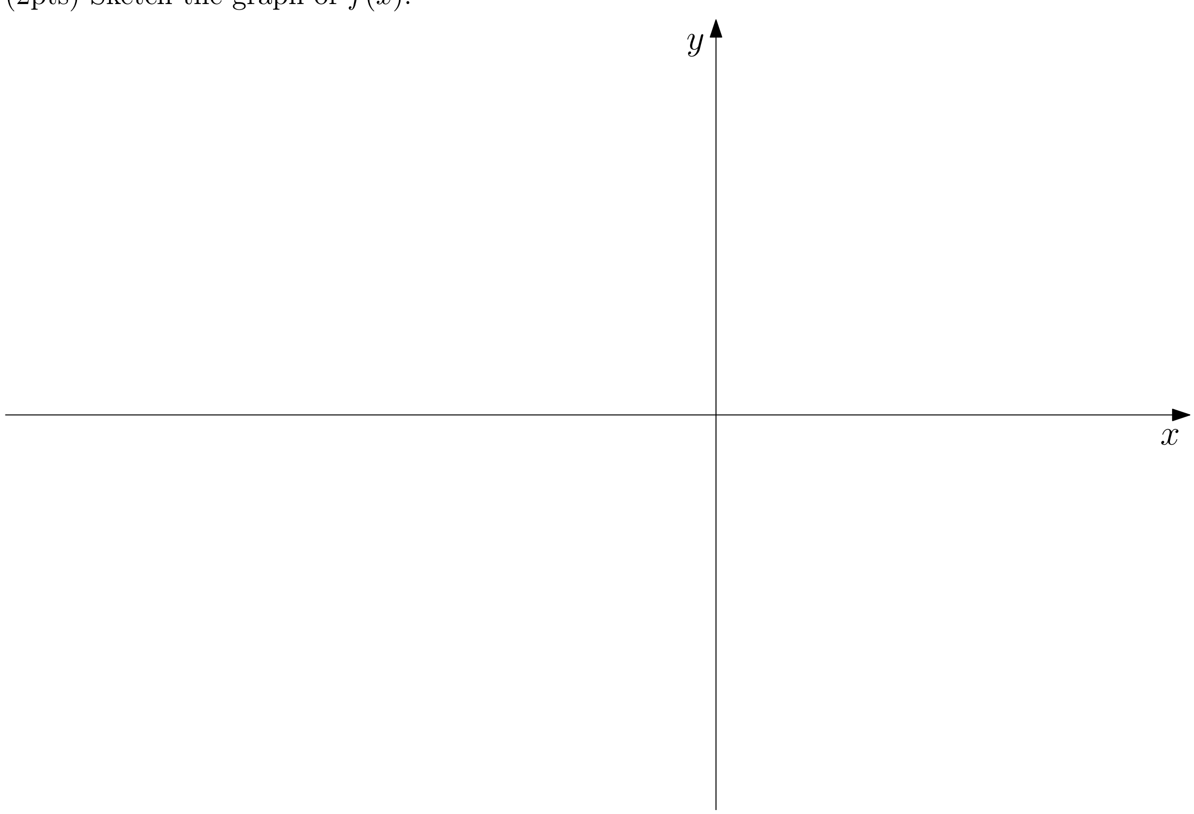
(d) (4pts) Find intervals on which $f(x)$ is concave up and intervals on which it is concave down. Find x-coordinates of inflection points of the graph of $f(x)$, if there are any.

(e) (2pts) Find the minimum value of $f(x)$ on the interval $[2, 5]$.

(f) (2pts) Sketch the graph of $f(x)$.



11. Evaluate the following integrals.

(a) (5pts) $\int_0^{\frac{\pi}{2}} 4\sin^2 x \cos x dx$

(b) (5pts) $\int \frac{6x+6x^2}{\sqrt{3x^2+2x^3}} dx$

(c) (5pts) $\int \left(5x^{\frac{2}{3}} + 4x(1-x^2) - \frac{2}{\sqrt[3]{x}}\right) dx$

12. In Lewis Carroll's book Alice's Adventures in Wonderland, there is a part where Alice falls down a well, but fortunately she slows down as she is falling. Suppose that our reference time point is such that at $t = 0$ Alice is 25 ft above the bottom of the well. Assume also that her speed at that moment is 10 ft/sec, and is decreasing at the rate of 2 ft/sec^2 from that moment on.

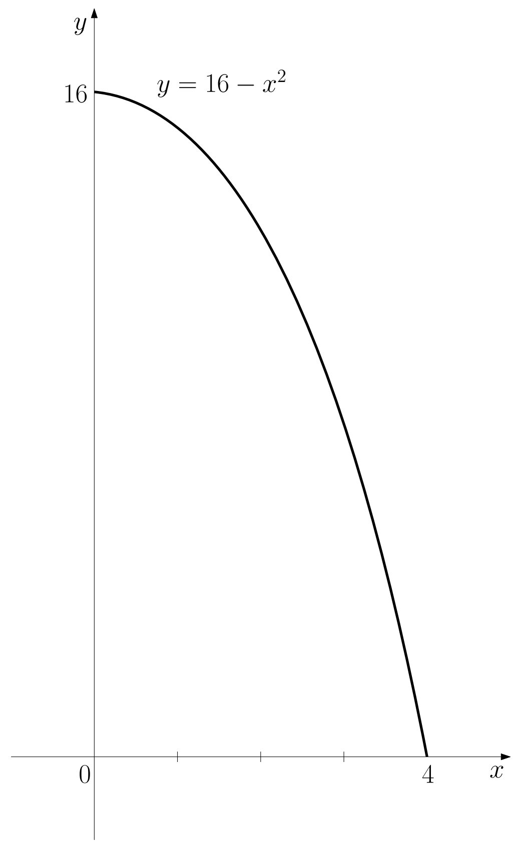
   (a) (4pts) Find the function that describes Alice's position at time t for $t \geq 0$.

   (b) (2pts) After how many seconds will Alice reach the bottom of the well?

   (c) (2pts) What will Alice's speed be when she reaches the bottom?

13. Consider the function $f(x) = 16 - x^2$ pictured below.

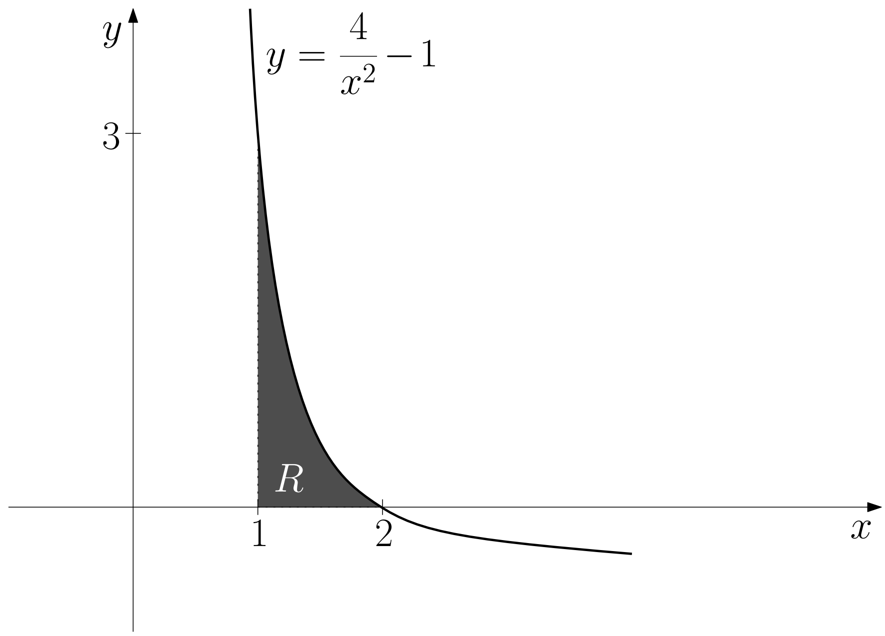
(a) (6pts) Estimate $\int_0^4 f(x)dx$ with a Riemann sum using four subintervals of equal width and left endpoints.



(b) (2pts) Sketch the rectangles that you used in part (a) on the provided graph.

(c) (2pts) Find the exact value of $\int_0^4 f(x)dx$.

14. Consider the region R bounded by the x-axis, the curve $y = \dfrac{4}{x^2} - 1$, and the line $x = 1$.



(a) (6pts) Find the area of the region R.

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(b) (4pts) Set up **but do not evaluate** the integral that gives the volume of the solid obtained by revolving the region R about the x-axis.

(c) (4pts) Set up **but do not evaluate** the integral that gives the volume of the solid obtained by revolving the region R about the y-axis.