Final Exam Math 241 Spring 2020

Name:
Section Number:

Read all of the following information before starting the exam:

- The exam begins on Wednesday, May 13 as 12:00. You have 2 hours for the exam and 30 minutes to prepare the exam to be submitted. Please make sure to submit your exam on Laulima under "Tests and Quizzes" by 2:30pm HST.
- Books and notes may be used. No calculators, unauthorized websites or communication are allowed during the exam.
- Show all work, clearly and in order using proper notations, if you want to get full credit. We reserve the right to take off points if we cannot see how you arrived at your answer (even if your final answer is correct).
- This test has 15 pages total, including this cover sheet, and is worth 100 points. It is your responsibility to make sure that you have all of the pages!

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1. 8pts Multiple Choice (2 points each)

   1. Evaluate the following limit

   $$\lim_{x \to 2^+} \frac{|2(x-2)(x-3)|}{(x-2)}$$

      a. 0
      b. 2
      c. -2
      d. Does not exist.

   2. Evaluate the following derivative (answer may not be simplified)

   $$\frac{d}{dx}(x\sqrt{x^2+3})$$

      a. $\sqrt{2x}$
      b. $\frac{1}{2}(x^2+3)^{-\frac{1}{2}}(2x)$
      c. $\frac{2x+3}{\sqrt{x^2+3}}$
      d. $\sqrt{x^2+3} + \frac{x}{2}(x^2+3)^{-\frac{1}{2}}(2x)$

   3. Evaluate the following derivative (answer may not be simplified)

   $$f(x) = \frac{x^2+2}{(x-1)}$$

      a. $f'(x) = \frac{2x(x-1)-x^2-2}{(x-1)^2}$
      b. $f'(x) = 2x$
      c. $f'(x) = \frac{x^2-2x+2}{(x-1)^2}$
      d. $f'(x) = \frac{2x+(x-1)(x^2+2)}{(x-1)^2}$

   4. Evaluate the following derivative (answer may not be simplified)

   $$g(t) = \tan^2(t+\pi)$$

      a. $g'(t) = 2\tan(t)\sin^2(t)$
      b. $g'(t) = 2\tan(t+\pi)\sec^2(t+\pi)$
      c. $g'(t) = 2\tan(t)$
      d. $g'(t) = 2t\sec^2(t+\pi)$

2. 6pts Compute the following limits (including $+\infty$ or $-\infty$) or show that they do not exist. You must show some work for each problem to receive full credit. You may not use L'Hopital's Rule.

a.

$$\lim_{x\to\infty} \frac{x-1}{x^2-3x+2}$$

b.

$$\lim_{t\to 0} \frac{\sin(3t)}{t}$$

c.

$$\lim_{x\to 4^-} \frac{\sqrt{x^6+1}}{(x-4)}$$

3. 6pts **Using the definition of derivative,** show that $f'(2) = \frac{1}{4}$ for

$$f(x) = \sqrt{x+2}.$$

4. 6pts Find the derivatives ($\frac{dy}{dx}$) of the following functions

a.

$$y = \int_2^{\sin x} \sqrt{t^2+1}\, dt$$

b.

$$y^3 + x\cos(y) = 8.$$

5. 12pts Evaluate the following integrals. **Remember to show all steps to receive full credit.**

   1. $$\int \sec^2(x)\tan^3(x)dx$$

   2. $$\int_{-1}^{1} t(t+3)\,dt$$

   3. $$\int x\sqrt{x+4}dx$$

   4. $$\int_0^2 \frac{1}{(2x+1)^3}dx$$

6. 5pts Find the area enclosed between the line $y = 1$ and the curve $y = x^2 - 4x + 4$.

7. 6pts Consider the region R between the line $y = 1$ and the curve $y = x^2 - 4x + 4$ (as above). Set up but **do not evaluate** the integrals for the following.

   a. the volume of the region obtained by rotating R around the x-axis.

   b. the volume of the region obtained by rotating R around the y-axis.

8. 5 pts Water is flowing out of a tank at a rate (in liters per second) of

$$V'(t) = -4t.$$

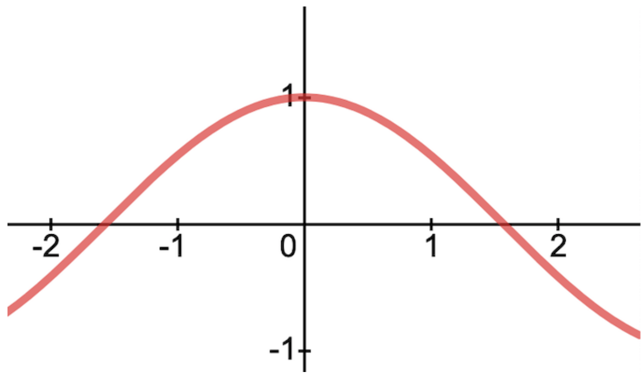
   a. The tank has 18 liters at time $t = 0$. Find an equation for $V(t)$.

   b. How long until the tank is empty?

9. 6pts We have 27 m^2 of material to make a box without a top. The base of the box should be a square and the four sides should be rectangles. Find the maximum volume of such a box.

10. 6pts A child is flying a kite. The kite is 30 ft above the ground and is moving horizontally away from the child at a rate of 5 ft per second. How fast is the distance between the child and the kite changing when 50 feet of string has been let out. (Simplify your answer).

11. 5pts Below is a graph of the function $y = \cos(x)$:



a. Use the left-endpoint rule with $n = 4$ to estimate $\int_0^{\frac{\pi}{2}} \cos(x)\, dx$. (You do not need to simplify your answer).

b. Plot the rectangles that you used for the estimation on the graph.

c. Is this estimate over or under the actual answer? Or is it impossible to tell? Explain your answer.

d. Fill in the following expression(both boxes) for the Riemann sum approximation for arbitrary n with left-end points

$$\int_0^{\frac{\pi}{2}} \cos(x)\, dx \approx \sum_{i=1}^{n} \underbrace{\boxed{}}_{\Delta x} \cdot \cos(\underbrace{\boxed{}}_{x_i^*}).$$

12. 6pts **The derivative** of a function $f(x)$ is given by

$$f'(x) = (x-2)(x+4)\sqrt{x^2+1}.$$

a. Find an equation for the tangent line at the point $(1, 6)$.

b. Give the intervals where the **function** $f(x)$ is increasing.

c. Find the x-coordinate of any local minima of $f(x)$.

13. 5pts Graph a function with the following properties

- $f'(x) > 0$ on the interval $(-1, 4)$ and $f'(x) \leq 0$ everywhere else.
- $f''(x) < 0$ on the interval $(0, 7)$ and $f''(x) \geq 0$ everywhere else.
- $\lim_{x\to\infty} f(x) = 3$.

14. 5pts Consider the function $f(x) = x^{13} + 9x - 4$.

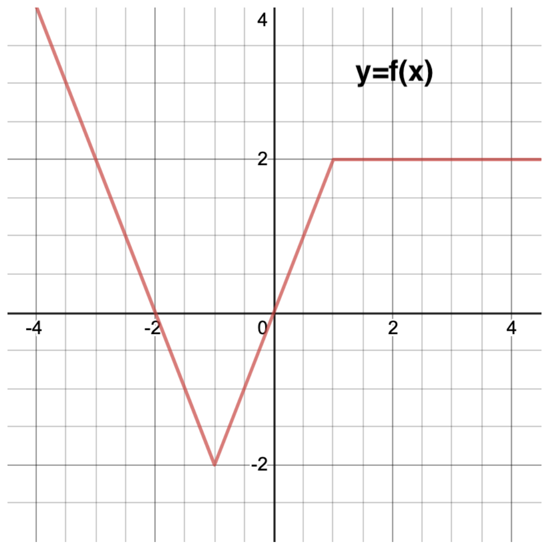
   a. Show that the function $f(x)$ has at least one solution $f(x) = 0$. State any theorems used.

   b. Use the Mean-value Theorem or Rolle's Theorem to show that there is **only one** solution $f(x) = 0$.

15. 5pts Use linear approximation or differential to approximate the value of $10^{\frac{1}{3}}$ using the fact that $8^{\frac{1}{3}} = 2$.

16. 8 pts Use the function f plotted below to define

$$g(x) = \int_0^x f(t)dt$$



Answer the following questions about the function $g(x)$.

a. Evaluate:
   - $g(0)$
   - $g(4)$
   - $g(-3)$

b. Evaluate:
   - $g'(-2)$
   - $g''(-2)$

c. On what interval(s) is the function $g(x)$ concave up (you may assume the lines continue to $\pm\infty$)?