

Name (Print): _____
Section Number: _____

[illegible]

Question 1. (7 points)

The movement of two pōpoki are tracked. Pōpoki Alikā has a velocimeter strapped to her, while the distance travelled by pōpoki Kapono is tracked via GPS.



Alikā



Kapono

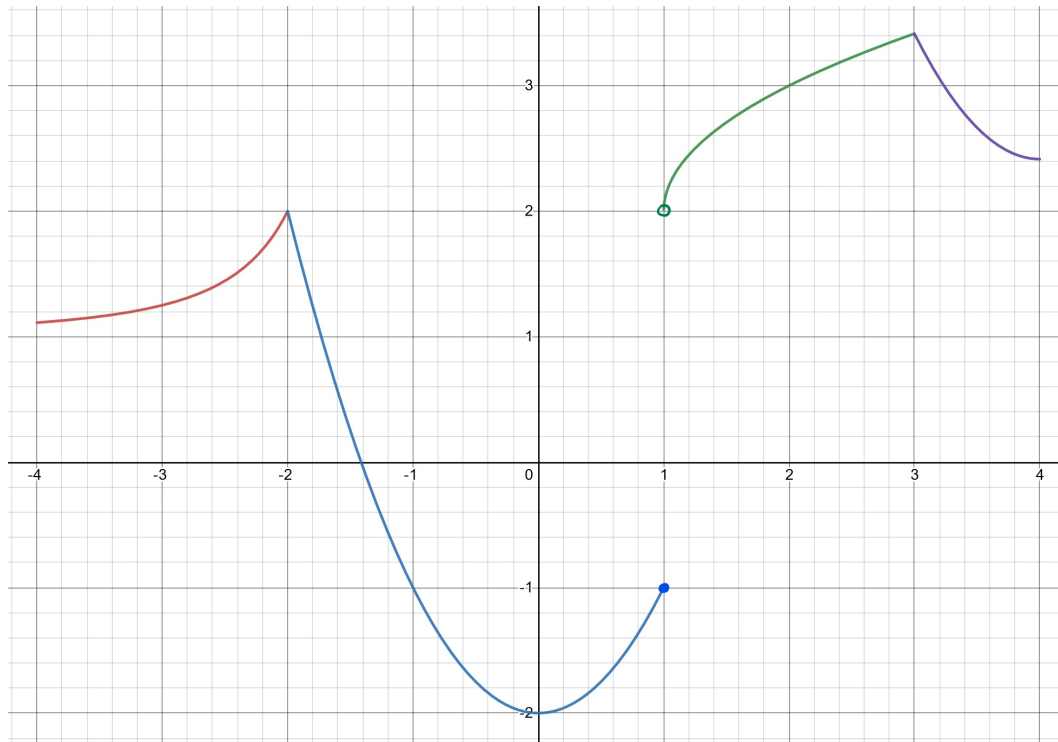
Both travel in a straight line from the same location. Their respective speed and distance travelled are recorded below.

Time in seconds	Speed of Alikā in meters per second	Distance traveled by Kapono in meters
0	0	0
2	2	2
4	2	10
6	6	22
8	12	28
10	13	32
12	10	40
14	4	44

- (a) (2 points) What is the average acceleration of Alikā from time 4 seconds and time 10 seconds?
- (b) (3 points) Estimate the distance traveled by Alikā over the 14 second period.
- (c) (2 points) From this data, does it seem likely that Alikā and Kapono stayed close together throughout these 14 seconds, or did they get far apart? Give a brief justification.

Question 2. (6 points)

Consider the function $f(x)$ with the graph $y = f(x)$ pictured below. The domain of f is $[-4, 4]$.



- (a) (1 point) On which interval(s) (if any) is the function decreasing?
- (b) (1 point) On which interval(s) (if any) is the function concave up?
- (c) (1 point) Where (if anywhere) is the function **not** continuous?
- (d) (1 point) Where (if anywhere) is the function **not** differentiable?
- (e) (1 point) What is $\lim_{x \rightarrow 1^+} f(x)$ (a reasonable estimate is OK / 'does not exist' is a possible answer)?
- (f) (1 point) What is $\lim_{x \rightarrow 3^-} f'(x)$ (a reasonable estimate is OK / 'does not exist' is a possible answer)?

Question 3. (9 points)

You are given that a function $f(x)$ satisfies the following limits:

$$\lim_{x \rightarrow -1} f(x) = 4, \quad \lim_{x \rightarrow 3^+} f(x) = -12, \quad \lim_{x \rightarrow \infty} f(x) = 7.$$

Use these to compute the following limits, or say if they are $+\infty$ or $-\infty$. Do **not** use l'Hôpital's rule.

(a) (3 points) $\lim_{x \rightarrow -1} \frac{xf(x) + 4x}{x^2 + 2}$

(b) (3 points) $\lim_{x \rightarrow 3^+} \frac{(f(x) + 6)^2}{x - 3}$

(c) (3 points) $\lim_{x \rightarrow \infty} \left(\sqrt{4x^2 + x} - 2x + f(x) \right)$

Question 4. (6 points)

Using the limit definition of the derivative, compute $f'(x)$ when $f(x) = \sqrt{6-x}$.

You will get no credit for computing the derivative without using the definition.

Question 5. (12 points)

For this question, let f be a differentiable function such that f and f' take the following values:

x	-1	0	1	2	3
$f(x)$	-4	-3	-2	2	0
$f'(x)$	1	2	-2	4	1

Compute the following derivatives. Simplify your answers as much as you can.

(a) (4 points) Compute $g'(3)$ if $g(x) = f(\sin(\pi x) - \cos(\pi x))$.

(b) (4 points) Compute $h'(2)$ if $h(x) = \frac{f(x)}{x^2 + 1}$.

(c) (4 points) Compute $j'(9)$ if $j(x) = f(f(\sqrt{x}))$.

Question 6. (8 points)

Let f be a differentiable function. Consider the equation

$$xy^2 - 2f(x)y = -3.$$

(a) (4 points) Compute y' . Your answer should be in terms of x , y , $f(x)$, and $f'(x)$.

(b) (4 points) You are also told that $f(1) = 2$ and $f'(1) = -3$. Find the equation of the tangent line to the curve at the point $(1, 1)$. Write the equation in the form $y = mx + b$.

Question 7. (6 points)

A circular cylinder with a top and bottom is growing in such a way that its height is always three times its diameter.

If the radius of the cylinder is changing at a rate of $\frac{1}{8}$ ft/min, find the rate at which the cylinder's volume is changing when the radius is 4 feet. (Hint: Use the formula $V = \pi r^2 h$.)

Question 8. (9 points)

Consider the function f defined on the interval $[0, 2\pi]$, whose derivative is given below:

$$f'(x) = \frac{2 \sin(x) - 1}{2 + \sin(x)}.$$

- (a) (2 points) On what subintervals (if any) of $[0, 2\pi]$ is f increasing and on which is it decreasing?
- (b) (2 points) Where (if anywhere) does f have a local minimum or a local maximum?
- (c) (5 points) On what subintervals (if any) of $[0, 2\pi]$ is f concave up or concave down, and where (if anywhere) does f have an inflection point?

Question 9. (9 points)

Let $f(x) = x^3 + x + \frac{1}{2}$.

- (a) (3 points) Use a linear approximation or differential to estimate how much f changes when x changes by 0.1 relative to $x = 0$.

- (b) (3 points) The function f must have at least one zero in the interval $(-1, 0)$. Explain why, making explicit which theorem(s), if any, and which assumption(s) on f , if any, you are using.

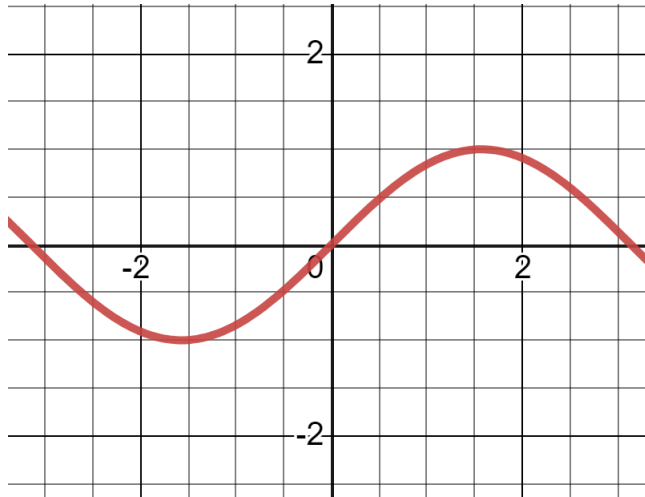
- (c) (3 points) The function f must have at most one zero in the interval $(-1, 0)$. Explain why, making explicit which theorem(s), if any, and which assumption(s) on f , if any, you are using.

Question 10. (6 points)

A rectangular box has a base that is a square. The perimeter of the base plus three times the height of the box is equal to 3 ft. What is the largest possible volume for such a box, and what are its dimensions? Justify your answer.

Question 11. (6 points)

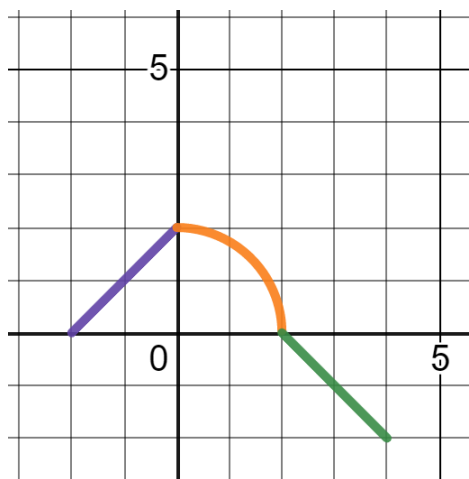
Below is a graph of the function $y = \sin(x)$:



- (a) (3 points) Use the right-endpoint rule with $n = 4$ to estimate $\int_0^{\pi/2} \sin x \, dx$. (No simplification is needed).
- (b) (1 point) On the figure above, or on a sketched copy of the graph, plot the rectangles that you used for the estimation.
- (c) (2 points) Is this estimate over or under the actual answer? Or is it impossible to tell? Explain your answer.

Question 12. (8 points)

Below is a graph of the function g . For x in $[-2, 4]$ we define $f(x) = \int_0^x g(t) dt$.



(a) (3 points) Find $f(0)$ and $f(4)$.

(b) (2 points) Find all critical points of the function f on the interval $(-2, 4)$.

(c) (3 points) Find the absolute maximum value of $f(x)$ on the interval $[-2, 4]$.

Question 13. (5 points)

The Fundamental Theorem of Calculus says the following. If f is a continuous function on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t) dt$$

is continuous on $[a, b]$ and differentiable on (a, b) , with $g'(x) = f(x)$.

A function $h(x)$ is defined for $x \in [0, \pi/2]$ by

$$h(x) = \int_{\cos x}^{\sin x} \sqrt{1-t^2} dt.$$

Find $h'(x)$ by using the above version of the Fundamental Theorem of Calculus.

You will get no credit if you do not use this version of the Fundamental Theorem of Calculus.

Question 14. (9 points)

a) (3 points) Solve the indefinite integral $\int (x^2 - 5 \sin x + \sec^2 x) \, dx$

b) (3 points) Solve the definite integral $\int_2^{23} \frac{1}{\sqrt{x+2}} \, dx$

c) (3 points) Solve the indefinite integral $\int \cos x \sin^2 x \sqrt{1 - \sin^3 x} \, dx$

Question 15. (6 points)

- a) (2 points) Graph and shade the area bounded by the following. Label points of intersection and endpoints of the graph.

$$y = |x|, y = x^3, -1 \leq x \leq 1$$

- b) (4 points) Solve for the area of the shaded region.

Question 16. (8 points)

Setup the integral but **DO NOT EVALUATE** the following volumes using the prescribed method.

- a) (4 points) The volume of the solid generated using the area bounded by

$$y = 0, \quad y = \sqrt{x^2 + 1}, \quad 0 \leq x \leq 5$$

rotated about the y-axis. Use the cylindrical shell method (setup but **do not solve** the integral).

- b) (4 points) The volume of the solid generated using the area bounded by

$$-y^2 + 2y = x, \quad x = 0$$

rotated about the y-axis. Use the disk method (also known as the washer method; setup but **do not solve** the integral).