## MATH 241 COMMON FINAL EXAM, SPRING 2022

You have 120 minutes.
No books, no notes, no electronic devices.
YOU MUST SHOW ALL WORK. NO NEED TO SIMPLIFY ANSWERS.

Name $\qquad$
Instructor Name $\qquad$
Section Number $\qquad$

Grade table (for instructor's use only)

1. (16pts) $\qquad$
2. $(4 \mathrm{pts}) \square$
3. (8pts) $\qquad$
4. $(20 \mathrm{pts})$
5. (5pts) $\qquad$
6. (8pts)
7. (8pts) $\qquad$
8. $(10 \mathrm{pts})$
9. $(24 \mathrm{pts}) \longrightarrow$ Total Score (/150 points)
$\qquad$
10. (8pts)
11. (10pts) $\qquad$
12. (14pts) $\qquad$
13. Calculate the following limits. Do not use L'Hospital's rule. If the limit is infinite, specify whether it is $\infty$ or $-\infty$.
(a) (4pts) $\lim _{x \rightarrow-2^{-}} \frac{x+2}{x^{2}+4 x+4}$
(b) (4pts) $\lim _{x \rightarrow 0^{+}} \sqrt{x^{2}+x} \sin (x)$
(c) (4pts) $\lim _{x \rightarrow-2} \frac{2-|x|}{2+x}$
(d) $(4 \mathrm{pts}) \lim _{x \rightarrow \infty}\left(\sqrt{9 x^{2}+x}-3 x\right)$
14. Consider the function $f$ defined by

$$
f(x)= \begin{cases}-x & \text { if } x<-1 \\ \sin \left(\frac{\pi}{2} x\right) & \text { if }-1 \leq x<1 \\ 2 & \text { if } x=1 \\ 2-x & \text { if } x>1\end{cases}
$$

(a) (2pts) Sketch the graph of $f$.

(b) (1pt) Find the values $a$ such that $\lim _{x \rightarrow a} f(x)$ does not exist. No justification needed.
(c) (1pt) Find the values $a$ such that $f(x)$ is discontinuous at $x=a$. No justification needed.
3. Consider the function $f(x)=\frac{1}{\sqrt{x}}$.
(a) (4pts) Using the definition of the derivative as a limit, compute $f^{\prime}(x)$. (Warning: you will not get credit if you use the rules of differentiation.)
(b) (2pts) What is the domain of $f^{\prime}$ ?
(c) (2pts) Find the equation of the tangent line to the curve $y=f(x)$ at the point $(4,1 / 2)$.
4. In each of the following, calculate the derivative $\frac{d y}{d x}$. You do not need to simplify your answers.
(a) (5pts) $y=\frac{x^{2}+2}{x^{5}+3}$
(b) (5pts) $y=\sqrt{x} \cos \left(x^{2}\right)$

In each of the following, calculate the derivative $\frac{d y}{d x}$. You do not need to simplify your answers.
(c) $(5 \mathrm{pts}) y=\left(\sqrt{x}+\frac{2}{x}\right)^{7}$
(d) (5pts) $y=\int_{0}^{1 / x} \sin ^{4} t d t$
5. (5pts) Find the slope of the tangent line to the graph of $x^{3}-3 x^{2} y+2 x y^{2}=0$ at the point $(1,1)$.
6. Consider the equation $2 x-1=\sin x$.
(a) (4pts) Explain why the equation has a solution in the interval $[0, \pi / 2]$. You may use the Intermediate Value Theorem.
(b) (4pts) Explain why the equation cannot have more than one solution in the interval $[0, \pi / 2]$. You may use Rolle's Theorem or the Mean Value Theorem.
7. ( 8 pts ) A spherical snowball melts so that its surface area decreases at a rate of $1 \mathrm{~cm}^{2}$ per minute. Find the rate at which the diameter decreases when the radius is 5 cm .
(Recall that the surface area of a sphere of radius $r$ is $4 \pi r^{2}$.)
8. (10pts) A box with a square base and open top must have a volume of $32 \mathrm{~cm}^{3}$. Find the dimensions of the box that minimize the amount of material used.
9. Let $f(x)=\frac{1}{x^{2}-1}$. Then $f^{\prime}(x)=-\frac{2 x}{\left(x^{2}-1\right)^{2}}$ and $f^{\prime \prime}(x)=\frac{6 x^{2}+2}{\left(x^{2}-1\right)^{3}}$ (you may take those formulas for granted).
(a) (2pts) Find the domain of $f$.
(b) (2pts) Find the intercepts with the $x$ and $y$-axes, if there are any.
(c) (2pts) Find the vertical asymptotes of $f$, if there are any.
(d) (2pts) Find the horizontal asymptotes of $f$, if there are any.
(e) (4pts) Find the intervals on which $f$ is increasing and the intervals on which $f$ is decreasing.
(f) (4pts) Find the local minimum values and the local maximum values, if there are any.
(g) (4pts) Find the intervals on which $f$ is concave up, and the intervals on which $f$ is concave down.
(h) (2pts) Identify all inflection points, if there are any.
(i) (2pts) Sketch the graph of $f$.

10. Evaluate the following integrals.
(a) $(5 \mathrm{pts}) \int_{0}^{\sqrt{\pi}} x \sin \left(x^{2}\right) d x$
(b) $(5 \mathrm{pts}) \int \frac{x^{3}}{\left(x^{4}-5\right)^{2}} d x$
(c) (5pts) $\int_{0}^{2}|x-1| d x$
11. ( 8 pts ) A particle moves in a straight line and has acceleration given by $a(t)=6 t+4$. Its initial velocity is $v(0)=-6 \mathrm{~cm} / \mathrm{s}$ and its initial position is 9 cm in the positive direction from the origin. Find the position of the particle after 2 seconds.
12. Consider the parabola $y=x^{2}-1$ between $x=1$ and $x=4$, pictured below.

(a) ( 6 pts ) Estimate the area under the parabola and above the $x$-axis between $x=1$ and $x=4$ with a Riemann sum, using three subintervals of equal width and right endpoints.
(b) (2pts) Sketch the rectangles that you used in part (a) on the provided graph.
(c) $(2 \mathrm{pts})$ Is your answer in (a) larger or smaller than the true area $\int_{1}^{4}\left(x^{2}-1\right) d x$ ? Explain.
13. Consider the region $R$ in the first quadrant bounded by the curves $y=x^{4}$ and $y=3 x-2 x^{2}$, pictured below. The two curves intersect at the points $(0,0)$ and $(1,1)$.

(a) (6pts) Find the area of the region $R$.
(b) (4pts) Set up but do not evaluate an integral for the volume of the solid obtained by rotating $R$ about the $x$-axis.
(c) (4pts) Set up but do not evaluate an integral for the volume of the solid obtained by rotating $R$ about the $y$-axis.

