Fall 2008

Math 100 – Survey of Mathematics (3)

Selected topics designed to acquaint nonspecialists with examples of mathematical reasoning. May not be taken for credit after 215 or higher.

**Syllabus**. Math 100 is a terminal course covering topics from the major areas of study that make up mathematics. (Warn students that this course does **NOT** prepare them for Math 140 or any other mathematics course.)

A list of five areas of study, together with some subsidiary topics for each area, is given below. Individual instructors may wish to modify the list to accommodate the strengths and weaknesses of the textbook they are using.

1. Number Theory
   a. Primes and composites: the number of primes is infinite. The number of consecutive composites is unbounded.
   b. The Fundamental Theorem of Arithmetic
   c. Greatest common divisors. Euclidean algorithm.
   d. Perfect numbers
   e. Pythagorean triples
   f. Goldbach's conjecture
   g. Congruence with application (e.g. "casting out nines")
2. Topology
   a. Graph and network theory (e.g. "Königsberg bridge problem")
   b. Euler number of polygons and polyhedra with applications: the five Platonic solids, the classification of closed surfaces, and the vector field problem
   c. Orientability (e.g. Möbius strip and Klein bottle are non–orientable surfaces)
3. Probability and Statistics
   a. Counting techniques: $n$–tuples, permutations, and combinations
   b. Probability space: sample space, event, and probability
4. Logic
   a. Elements of symbolic logic: statements and connectives
   b. Implication statement: inverse, converse, and contrapositive
   c. Types of arguments: direct, indirect, and syllogism

5. History of Mathematics
   a. Biographical sketches of the lives of some of the famous mathematicians. These sketches should probably be given *during the time the particular mathematician's area is being discussed.* Examples: Galois (algebra), Fermat (number theory), Euler (topology), Riemann (analysis), Bernoulli (probability and statistics), and Boole (logic).
   b. History of the famous problems in mathematics: solving equations with radicals, the Poincáre conjecture

Instructors may organize the course around the theme: "What do mathematicians do?" Another approach is to emphasize the following themes:

1. Mathematics and modern culture
2. Highlights in the history of mathematics
3. Vignettes on the practical import of mathematics
4. The mathematical method and problem solving

(Sources: M. Kline's *Mathematics in Western Culture*, E. T. Bells' *History of Mathematics*, A. Renyl's *Dialogues on Mathematics*, "Scientific American" and the text.)

**Grading.** Course grades are based on three examinations during the semester and a final examination.

**Suggested Textbooks**: Miller, Charles D. and Vern E. Heeren, *Mathematical Ideas* or Stein, S., *Mathematics, the Man–Made Universe.*

**Course Objectives and Student Learning Outcomes**. Math 100 is a topics course, and the collection of topics covered depends on the instructor. A successful student will understand the material and will be able to apply it.

**Program Objectives**. The course is approved for FS credit, and the instructor shall strive to make sure the course meets the FS hallmarks.

1. Expose students to the beauty, power, clarity and precision of formal systems.
2. Help students understand the concept of proof as a chain of inferences.
3. Teach students how to apply formal rules or algorithms.
4. Require students to use appropriate symbolic techniques in the context of problem solving, and in the presentation and critical evaluation of evidence.
5. Not focus solely on computational skills.
6. Build a bridge from theory to practice and show students how to traverse this bridge.