Topology – Math 421 (3)

Course Description: General topology, including compactness and connectedness; the Jordan Curve Theorem and the classification of surfaces; first homotopy or homology groups

Prerequisite: Math 321 (or consent).

Textbooks: An excellent textbook is *Topology* by James R. Munkres, Prentice Hall (free online).

Topics:

- Topological Spaces and Continuity (definition of a topology, basis and subbasis of a topology, metric topologies, subspace topology, product topology, quotient topology, . . .)

- Connectedness and Compactness

- Countability and Separation Axioms

- Tychonoff Theorem

- Metrization Theorems

- Fundamental Group (Homotopy of paths, fundamental group, covering spaces, fundamental group of the circle, Borsuk–Ulam Theorem, Deformation retracts and homotopy type, fundamental group of some surfaces.)

- First homology group (optional). It would not be unreasonable to skip the fundamental group and covering spaces, introduce homology, and use H_1 as one of the discrete invariants in the classification of surfaces.

- Jordan curve theorem, Invariance of Domain, classification of surfaces.

Course objectives and Student Learning Outcomes. Upon successful completion of Math 421, the student will have a basic understanding of and will be able to use the key concepts from general topology, the idea of a topology, functorial constructions based on topologies, separation axioms,

and metrization. Students will also learn how to use discrete invariants to characterize geometric objects.

Program Objectives. Students should gain a basic understanding of the key concepts from general topology that are foundational to many areas in mathematics. Students should develop an appreciation for how discrete invariants, such as the Euler characteristic and fundamental group, are used in the classification of geometric objects, such as surfaces. In addition, students should further develop their ability to read and write proofs, and with this their understanding of proof as the scientific method of mathematics.