Math 442 (3 credits)

Course description: Vector operations, wedge product, differential forms, and smooth mappings. Theorems of Green, Stokes, and Gauss, both classically and in terms of forms. Applications to electromagnetism and mechanics.

Prerequisite: Math 244 or Math 253A, and Math 307 or Math 311, or consent.

Course objectives: Introduce students to a more general and rigorous treatment of the topics from multivariable calculus, and explain to students the interactions between linear algebra and analysis in multiple dimensions. Introduce students to manifolds and integration over them.

The course does not have Math 321 or 331 as a prerequisite, so the instructor need not attempt to give detailed proofs of all results. However, the course is a 400-level mathematics course: justifications of why results are true are expected, at least on a heuristic level.

Syllabus: The course should cover the following topics.

- Metric topology of Euclidean space, including the Heine-Borel theorem and completeness.
- Multilinear algebra sufficient to understand the later topics.
- Differentiable mappings between open subsets of Euclidean spaces, with the derivative treated as a linear map.
- Hessians and the second derivative test for maxima and minima.
- The inverse and implicit function theorems.
- Submanifolds of Euclidean space, and their tangent bundles.
- Optimization via Lagrange multipliers.
• Differential forms.
• Orientations and integrals of differential forms.
• Stokes’ theorem for submanifolds of Euclidean space.
• Applications to physics, for example Maxwell’s equations, mechanics, or special relativity.

Suggested textbooks:
• Spivak, *Calculus on manifolds*.
• Munkres, *Analysis on manifolds*.

Student Learning Outcomes: A successful student will:
• Understand the relationship between multivariable calculus and linear algebra.
• Understand how the general form of Stoke’s theorem relates to calculus topics, and be able to apply it.
• Gain a geometric appreciation of manifolds, orientations, and integration.

Program objectives: This course is designed partly to fulfil the following program learning objectives.
• Learn understand and be able to apply calculus in one and several variables.
• Learn understand and be able to apply linear algebra and vector spaces.
• Learn understand and be able to apply several mathematics topics at the junior and senior level.
• Formulate definitions and give examples and counterexamples.
• Apply mathematics to other fields.