

Syllabus for MATH 654

Graduate Introduction to Logic

October 7, 2017

1 Purpose and Objectives

Mathematical logic is one of the five (5) subareas within mathematics that graduate students study on the way to satisfying the requirements for MA or PhD in the Department of Mathematics. We have two courses at the specialized level, MATH 655 and MATH 657, as well as a graduate seminar focused on student presentations and covering rotating topics (MATH 649B). MATH 654 is a common introductory course in mathematical logic for graduate students, to be taken before MATH 655 and/or MATH 657.

2 Contribution to Learning Objectives

This course will strengthen graduate students' understanding of mathematics as a formal science and prepare them for research in the field. The course will prepare graduate students for research in mathematical logic, and in other fields that use mathematical logic. After taking 654 students should be

- well versed in the core results of mathematical logic,
- able to solve routine to medium level proof-based exercises, and
- prepared to enroll in one of the following existing graduate courses:
 - MATH 655 (Set Theory).
 - MATH 657 (Recursive Functions & Complexity).

3 Number of Credits, Level, and Prerequisites

Senior-level course in mathematical logic such as MATH 454–455 constitute excellent background for the course. However, graduate students in mathematics or closely related fields such as theoretical computer science are assumed to have the mathematical maturity required for the course, even if they have not studied mathematical logic at the senior level.

4 Course Syllabus

4.1 Course number and title

MATH 654: Graduate Introduction to Logic.

4.2 Description

MATH 654 is a lecture course meeting for three contact hours per week. 14 weeks are devoted to topics described below, and 1 week allotted for midterm exams and review.

4.3 Objectives

After taking this course, students will be fluent in the major topics of mathematical logic.

4.4 Student learning outcomes

The course will prepare graduate students for research in mathematical logic, and in other fields that use mathematical logic. After taking 654 students should be

- well versed in the core results of mathematical logic,
- able to solve routine to medium level proof-based exercises, and
- prepared to enroll in one of the following graduate courses:
 - MATH 655 (Set Theory).
 - MATH 657 (Recursive Functions & Complexity).

4.5 Number of credits

MATH 654 is a substantial lecture course with three contact hours per week; it thus counts for three credits.

4.6 Prerequisites

Graduate standing, or MATH 454 and MATH 455, or consent.

4.7 Textbooks

Kenneth Kunen: The Foundations of Mathematics

4.8 Grading and evaluation

Students may be graded on the basis of midterms, a final exam, and homework sets.

4.9 Schedule

1. **Introduction (2 weeks)**: syntax and semantics of first order logic.
2. **Model theory (4 weeks)**: Completeness, and Compactness theorems, Löwenheim-Skolem theorems.
3. **Set theory (3 weeks)**: axioms of set theory; ordinals and cardinals.
4. **Computability theory (4 weeks)**: Incompleteness theorem; computable and computably enumerable sets.
5. **Additional topics (1 week)**: description logics, modal logic, or other topics of current interest.

5 Relation to curriculum

In addition to its central place in the mathematical logic curriculum for graduate students, MATH 654 will be of interest to students in other areas of mathematics and science and to sufficiently prepared students in other fields such as philosophy. Model theory has strong connections with algebra. Axioms of set theory such as Choice (Zorn's Lemma) are widely used in mathematics. And more and more, mathematicians are concerned with the computability of their constructions. (The concern with *efficient* computability is also important, but will be covered in MATH 657.)