The story of the parallel axiom

We exclusively deal with plain geometry here and two lines are parallel if and only if they do not intersect or are equal.
(1) **Statements equivalent to Euclid’s Parallel Axiom**

(a) (John Playfair (1748-1819)) Given a line $\ell$ and a point $P$ not on $\ell$ there is at most one line parallel to $\ell$ and passing through $P$.

(b) There exists a triangle whose angle sum equals two right angles.

(c) There exist two triangles that are similar but not congruent.

(d) There exists a line $\ell$ such that the locus of all points on one side of $\ell$ with equal distance from $\ell$ is a straight line.

(e) Every triangle has a circumscribed circle.
(2) Attempts to prove the Parallel Axiom
(a) Ptolemy (85?-165?)
(b) Nasir al-din (1201-1274)
(c) Girolamo Saccheri (1667-1733) “Euclides ab omni naevo vindicatus”
(d) Johann Heinrich Lambert (1728-1777)
(e) Adrien-Marie Legendre (1752-1833)
(3) **Discoverers of Non-Euclidean Geometry**
   (a) Carl-Friedrich Gauss (1777-1855)
   (b) Janos Bolyai (1802-1860)
   (c) Nicolai Ivanovitch Lobachevsky (1793-1856), University of Kasan

(4) **Geometries galore**
   (a) Bernhard Riemann (1826-1866)
(5) **Consistency Proofs**

(a) Eugenio Beltrami (1835-1900)
(b) Arthur Cayley (1821-1895)
(c) Felix Klein (1849-1925)
(d) Henri Poincaré (1854-1912)