

The story of the parallel axiom

We exclusively deal with plain geometry here and two lines are **parallel** if and only if they do not intersect or are equal.

(1) **Statements equivalent to Euclid's Parallel Axiom**

- (a) (John Playfair (1748-1819)) Given a line ℓ and a point P not on ℓ there is at most one line parallel to ℓ and passing through P .
- (b) There exists a triangle whose angle sum equals two right angles.
- (c) There exist two triangles that are similar but not congruent.
- (d) There exists a line ℓ such that the locus of all points on one side of ℓ with equal distance from ℓ is a straight line.
- (e) Every triangle has a circumscribed circle.

(2) **Attempts to prove the Parallel Axiom**

- (a) Ptolemy (85?-165?)
- (b) Nasir al-din (1201-1274)
- (c) Girolamo Saccheri (1667-1733) “Euclides ab omni naevo vindicatus”
- (d) Johann Heinrich Lambert (1728-1777)
- (e) Adrien-Marie Legendre (1752-1833)

(3) **Discoverers of Non-Euclidean Geometry**

(a) Carl-Friedrich Gauss (1777-1855)

(b) Janos Bolyai (1802-1860)

(c) Nicolai Ivanovitch Lobachevsky (1793-1856), University of Kasan

(4) **Geometries galore**

(a) Bernhard Riemann (1826-1866)

(5) **Consistency Proofs**

(a) Eugenio Beltrami (1835-1900)

(b) Arthur Cayley (1821-1895)

(c) Felix Klein (1849-1925)

(d) Henri Poincaré (1854-1912)