Indefinite Integrals

The indefinite integral of $f$ with respect to $x$ is the family of all antiderivatives of $f(x)$. You can always check if some $F(x)$ is an antiderivative of $f(x)$ by differentiating $F$. Recall, any two antiderivatives of $f(x)$ differ by a constant. (e.g. $x^2$, and $x^2 + 3$ are both antiderivatives of $2x$.)

Find the indefinite integrals:

1. $\int x^{-3}(x + 1)\,dx$
2. $\int \frac{4 + \sqrt{t}}{t^3}\,dx$
3. $\int 3 \cdot \cos(5\theta)\,d\theta$
4. $\int -\frac{\sec^2(\tau)}{3}\,d\tau$
5. $\int 1 - \tan^2(\alpha)\,d\alpha$

Initial Value Problems

Finding an antiderivative of $f(x)$ is equivalent to solving the differential equation $\frac{dy}{dx} = f(x)$ (other words, find a $y(x)$ so that $y'(x) = f(x)$). A differential equation with an initial condition is called an initial value problem.

1. $\frac{dy}{dx} = 4x^3 + 3$ i.c $y(0) = 2$
2. $\frac{dy}{dx} = \frac{t^3 + t^2 - 1}{t^2}$ i.c $y(1) = 0$