Newton-Raphson Method

Newton’s method is a method for finding approximate roots of a function \( f(x) \). We will need \( f'(x) \) so the function must be differentiable. We begin by making a first guess of the root, call it \( x_0 \). We then compute \( x_1 \) by:

\[
x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}
\]

We continue this process until we are close enough for our application:

\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
\]

Use Newton’s method to find a root of \( x^3 - 5 = 0 \).
Find the 4th root of 15.
Where do the curves \( y = x^5 \) and \( y = 10x - 2 \) intersect.

Sums

We can do sums in wxmaxima. For \( \sum_{i=n}^{m} f(i) \), write in maxima as \( \text{sum}(f(i), i, n, m) \). Try out some sums.

1. \( \sum_{i=1}^{8} 2 \ast i^2 + 1 \)

We can estimate the area under a curve by approximating with rectangles. If we want to estimate the area under \( f(x) \) on the interval \([a, b] \), we can take the sum

\[
\sum_{i=1}^{n} f(a + i(\Delta x))\Delta x (\text{the right endpoint sum}), \text{ where } \Delta x = \frac{b-a}{n}.
\]

(Or, we could work out a formula for left endpoint sums or for midpoint sums. These each give us a different estimate, but as \( n \) gets bigger, they get closer together.)

Find an approximation for the following functions on the given intervals.

1. \( f_1(x) = 2x^2 \) on \([0, 2], n = 4 \)
2. \( f_2(x) = x^2 - x^3 \) on \([-1, 1], n = 10 \)
3. \( f_3(x) = \cos(2x) \) on \([0, \pi/2], n = 8 \)
4. \( f_4(x) = \sqrt{1 - x^2} \) on \([0, 1], n = 12 \)

Note the width of the subintervals need not be the same.
E.g. let \( f(x) = 4 - x^2 \) on \([-2, 2] \), using \(-1/2, 1/2, \text{ and } 1\) to partition the interval. Use right hand values.