Math 244 Final Review

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Justify all your work. Answers without suitable justification will receive no credit. Notes, books, calculators and other external aids are forbidden during the exam.

Formulas:

- **Work/Flow**: \( \int \mathbf{F} \cdot \mathbf{T} \, ds = \int \mathbf{F} \cdot \mathbf{dr} = \int M \, dx + N \, dy + P \, dz \).

- **Flux**: \( \int_C \mathbf{F} \cdot \mathbf{n} \, ds = \oint_C M \, dy - N \, dx \) (\( C \) is a closed path, oriented counterclockwise).

- **Green’s Theorem**:
  1. \( \oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \iint_R \left( \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) \, dxdy \) (Normal Form)
  2. \( \oint_C \mathbf{F} \cdot \mathbf{T} \, ds = \iint_R \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \, dxdy \) (Tangential Form)

- **Stokes’ Theorem**: If \( \text{curl} \mathbf{F} = \nabla \times \mathbf{F} \) and \( C \) is the boundary of a surface \( S \), then
  \[
  \oint_C \mathbf{F} \cdot \mathbf{dr} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, d\sigma
  \]

- **Divergence Theorem**: \( \text{div} \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} \) and \( D \) is the region enclosed by the closed surface \( S \), then
  \[
  \iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma = \iiint_D \nabla \cdot \mathbf{F} \, dV
  \]
Problem 1. Find the area between $y = x^2$ and $x = y^2$ in the first quadrant.

Problem 2. Find the volume of the region below $z = \sqrt{x^2 + y^2}$, above $z = -\sqrt{x^2 + y^2}$, inside $x^2 + y^2 + z^2 = 4$ and outside $x^2 + y^2 + z^2 = 1$.

Problem 3. Consider the region $D$ in the first octant bounded by $z = 1$, $y = 2z$ and $z = x$. Set up an integral (but do not evaluate it) for finding the volume of $D$ using the order of integration $dzdydx$.

Problem 4. Integrate $f(x, y, z) = xy + z^2$ along the line segment between $(1, -1, 0)$ and $(2, 2, 3)$.

Problem 5. Consider the vector field $\mathbf{F} = 2xe^{yz}i + (zx^2e^{yz} + \cos(y))j + yx^2e^{yz}k$.

1. Find a potential function for $\mathbf{F}$.
2. Find the work done by $\mathbf{F}$ along the boundary of the triangle with vertices $(0, 1, 1)$, $(2, 1, -1)$ and $(-2, 3, 1)$ oriented counterclockwise.

Problem 6. Consider $\mathbf{F} = (x^2 + y^2)i - (xy + x^2)j$ and let $C$ be the boundary of the parallelogram bounded by $x = y - 1$, $x = y + 1$, $y = 0$ and $y = 2$.

1. Find the clockwise circulation of $\mathbf{F}$ along $C$.
2. Find the outward flux of $\mathbf{F}$ across $C$.

Problem 7. Consider the surface $S$ cut from the parabolic cylinder $z = 4 - y^2$ by the planes $x = 0$, $x = 1$ and $z = 0$.

1. Parameterize the surface $S$.
2. Find the outward flux of $\mathbf{F} = z^2i + xj + 3zk$ across $S$.

Problem 8. Calculate the flux of the curl of $\mathbf{F}$ across $S$ where $\mathbf{F} = (z - y)i + xj + e^z k$ and $S$ is the portion of the sphere $x^2 + y^2 + z^2 = 25$ inside the cylinder $x^2 + y^2 = 16$ and above the $xy$-plane.

Problem 9. Find the outward flux of $\mathbf{F} = yi + xyj + (-z)k$ across the surface of the solid $x^2 + y^2 \leq 4$ between $z = 0$ and $z = \sqrt{x^2 + y^2}$.