Problem 24. Show that $ZF \vdash (\exists y)(\forall x)(x \not\in y)$. (See Exercise II.11.15.)

Problem 25. Let $f : 2^{<\omega} \to 2^{<\omega}$ be such that for all $\sigma, \tau \in 2^{<\omega}$, if $\sigma \prec \tau$, then $f(\sigma) \prec f(\tau)$. Show that $g : 2^\omega \to 2^\omega$ given by $g(X)|f(X|n)| = f(X|n)$ for $X \in 2^\omega$ is well-defined and continuous using the topology on $2^\omega$ defined in Definition 6. (Note that $|f(\sigma)| \geq |\sigma|$.)

Problem 26. Prove that the topology on $2^\omega$ defined in Definition 6 is the same as the topology induced by the metric defined in Problem 23. (Note that the topology induced by a metric is the topology generated by the set of all open balls defined using the metric.)