Eleventh International Conference on

Computability, Complexity and Randomness

CCR ‘16

Honolulu, January 4–8, 2016

Contents

PROGRAM

Monday, 4 January 2016

9.45 **Opening**

9.50 **Linda Westrick** (Victoria University of Wellington)
Seas of squares with sizes from a co-enumerable set

10.40 **Coffee Break**

11.10 **Valentina Harizanov** (George Washington University)
Degrees of the isomorphism types of structures

11.40 **Jake Pardo** (Pennsylvania State University)
Gambling Against Some Odds

12.10 **Lunch Break**

14.00 **Lance Fortnow** (Georgia Tech)
Tutorial I
Bounding rationality with computation

14.50 **Adam Case** (Iowa State University)
Turing Reductions and Data Processing Inequalities for Sequences

15.20 **Coffee**

15.50 **Hayato Takahashi** (Gifu University)
An example of non-computable conditional probability for generalized Lambalgen's theorem

Tuesday, 5 January 2016

9.10 **Uri Andrews** (University of Wisconsin, Madison)
Connections between Computation and Geometry

10.00 **Coffee Break**

10.30 **Satyadev Nandakumar** (Indian Inst. of Tech., Kanpur)
Pseudorandomness and dimension

11.30 **Rodney G. Downey** (Victoria University of Wellington)
A minimal degree computable from a weakly 2-generic one

12.00 **Lunch Break**

14.00 **Lance Fortnow** (Georgia Tech)
Tutorial II

14.50 **Jeffrey O. Shallit** (University of Waterloo)
Separating Words
With Automata

15.40 **Coffee**

Wednesday, 6 January 2016

9.10 **Cristóbal Rojas** (Universidad Andrés Bello, Chile)
Computability and complexity of random perturbations of dynamical systems

10.00 **Coffee Break**

10.30 **Benoit Monin** (Paris Créteil)
A unifying approach to the Gamma question

11.00 **Gregory Igusa** (University of Notre Dame)
Generic computability and coarse computability

11.30 **Lance Fortnow** (Georgia Institute of Technology)
Tutorial III

Free Afternoon

Thursday, 7 January 2016

9.10 **Ron Peretz** (Bar-Ilan University, Israel)
Effective martingales with restricted wagers

10.00 **Coffee Break**

10.30 **Rutger Kuyper** (University of Wisconsin–Madison)
Preservation of randomness and genericity under symmetric differences

11.30 **Wolfgang Merkle** (University of Heidelberg, Germany)
Being low for K along sequences and elsewhere

12.00 **Lunch Break**

14.00 **Rupert Hölzl** (National University of Singapore)
Randomness for Computable Measures and Initial Segment Complexity

14.30 **Dan Turetsky** (Victoria University of Wellington)
The complexity of free abelian groups

15.20 **Coffee Break**

15.50 **Mariya Soskova** (Sofia University)
Definability and automorphisms in the enumeration degrees

Friday, 8 January 2016

9.30 **Andrew Bridy** (University of Rochester)
Automatic Sequences and
Curves over Finite Fields

10.00 **Kohtaro Tadaki** (Chubu University)
The Principle of Typicality

10.30 **Coffee Break**

11.00 **Lance Fortnow** (Georgia Tech)
Public Talk

11.50 Group photo

12.20 **Sandwiches**

ABSTRACTS

Uri Andrews

Connections Between Computation and Geometry

Abstract: The spectrum problem in computable model theory is to determine which sets $S \subseteq \omega + 1$ can occur as $\mathrm{Spec}(T)$, the set of dimensions of recursive models of T, for some strongly minimal theory T. Though we know very little about the spectrum problem in general, we now have a far better understanding in the case where T has an understandable geometry.

Andrew Bridy

Automatic Sequences and Curves over Finite Fields

Abstract: A theorem of Christol states that a power series y over the finite field $\mathbb{F}_q$ is an algebraic function if and only if its coefficient sequence can be produced by a finite automaton that reads the base-q expansions of positive integers. The proof uses combinatorics and linear algebra, but hidden in the theorem there is geometric information about a curve that contains y in its function field. I make this explicit by demonstrating a precise link between the complexity of the automaton and the geometry of the curve.

Adam Case

Turing Reductions and Data Processing Inequalities for Sequences

---

Abstract: A data processing inequality states that the quantity of shared information between two entities (e.g. signals, strings) cannot be significantly increased when one of the entities is processed by certain kinds of transformations. Various branches of information theory have developed their own data processing inequalities. For example, if $f : \{0,1\}^* -> \{0,1\}^*$ is a computable function, then there is a constant c such that, for all strings $x, y \in \{0,1\}*$,

$$I(f(x) : y) \leq I(x : y) + c,$$

where $I(x : y) = K(y) - K(y \mid x)$ is the algorithmic mutual information between x and y, and $K(y)$ and $K(y \mid x)$ are the Kolmogorov complexity of y and the conditional Kolmogorov complexity of y given x, respectively [3]. However, there are settings within algorithmic information theory that do not have known data processing inequalities.

In this talk, we discuss new data processing inequalities for sequences where transformations are represented by Turing reductions. To measure the shared information between two sequences, we use a recent advancement in constructive dimension called mutual dimension [1]. Formally, the mutual dimension between two sequences S and T is defined by

$$\mathrm{mdim}(S : T) = \liminf_{n \to \infty} \frac{I(S[0..n-1] : T[0..n-1])}{n}.$$

Intuitively, this is the density of algorithmic mutual information between S and T [2]. We show how adjustments to the computable bounds on the use function of a Turing reduction will yield different data processing inequalities.

[1] Adam Case and Jack H. Lutz. Mutual dimension. *ACM Transactions on Computation Theory*, 7(12): 1-26, 2015.
[2] Adam Case and Jack H. Lutz. Mutual dimension and random sequences. In *Proceedings of the Fortieth International Symposium on Mathematical Foundations of Computer Science*, Part II, pages 199-210. Springer, 2015.
[3] Ming Li and Paul Vitanyi. *An Introduction to Kolmogorov Complexity and Its Applications*. Springer, third edition, 2008.

Rodney G. Downey

A minimal degree computable from a weakly 2-generic one

Joint work with Satyadev Nandakumar.

Abstract: We prove that such objects exist, solving a question of Barmpalias and Lewis-Pye. The result is right on the edge of what is possible since Jockusch proved that 2-generics are downward dense below degrees below 2-generics. Moreover, any weakly 2-generic degree will form a minimal pair with $\mathbf{0}'$. Thus, whilst the construction is a full approximation one, both degrees will be in $\Delta^0_3 - \Delta^0_2$, causing some complexity.

Lance Fortnow

Bounding rationality with computation

Abstract: Traditional microeconomic theory treats individuals and institutions as completely understanding the consequences of their decisions given the information they have available. These assumptions might require these agents to solve hard computational problems to optimize our choices. What happens if we restrict the computational power of economic agents?

There has been some work in economics treating computation as a fixed cost or simply considering the size of a program. This series of talks, based on the work of the speaker and others, will explore a new direction bringing the rich tools of computability and computational complexity into economic models.

We show how to incorporate computability and computational complexity into a number of economic models including game theory, prediction markets, forecast testing, preference revelation and awareness. We will suggest a number of further directions worth exploring.

We will not assume any background in economics or computational complexity.

Valentina Harizanov

Degrees of the isomorphism types of structures

Abstract: The Turing degree spectrum of a countable structure A is the set of all Turing degrees of the isomorphic copies of A. Knight proved that the degree spectrum of a structure is closed upwards, unless the structure is automorphically trivial, in which case the degree spectrum consists of a single Turing degree. Hirschfeldt, Khoussainov, Shore, and Slinko established that for every automorphically nontrivial structure A, there is a symmetric irreflexive graph, a partial order, a lattice, a ring, an integral domain of arbitrary characteristic, a commutative semigroup, or a 2-step nilpotent group the degree spectrum of which coincides with the degree spectrum of A.

Jockusch and Richter defined the degree of the isomorphism type of a structure A to be the least Turing degree in the degree spectrum of A. Such degrees may not exist. For example, Richter showed that linear orders and trees without computable isomorphic copies do not have degrees of their isomorphism types. A.N. Khisamiev established the same result for abelian p-groups. Richter also introduced a general combination method for building structures the isomorphism types of which have arbitrary Turing degrees. As a corollary, she showed that there is an abelian torsion group with an arbitrary degree of its isomorphism type. We will show how Richter's combination method can be extended. We will present some recent results on the degrees of the isomorphism types of structures of interest in algebra, geometry, and low-dimensional topology. These results are obtained in collaboration with different groups of researchers.

Rupert Hölzl

Randomness for Computable Measures and Initial Segment Complexity

Joint work with Christopher P. Porter (http://arxiv.org/abs/1510.07202).

Abstract: The Levin-Schnorr theorem establishes the equivalence of a certain measure-theoretic notion of typicality for infinite sequences (known as Martin-Löf randomness) with a notion of incompressibility given in terms of Kolmogorov complexity. Although the Levin-Schnorr theorem is usually formulated for sequences that are random with respect to the Lebesgue measure on 2^ω, it is well known that the theorem can be generalized to hold for any computable probability measure on 2^ω. More specifically, a sequence $X \in 2^\omega$ is Martin-Löf random with respect to a computable measure μ if and only if the initial segment complexity of $X \upharpoonright n$ is bounded from below by $-\log \mu(X \upharpoonright n)$. Thus we see that certain values of the measure μ constrain the possible values of the initial segment complexities of the μ-random sequences.

In this study, we further explore the interaction between computable measures and the initial segment complexity of the sequences that are random with respect to these measures (hereafter, we will refer to those sequences that are random with respect to a computable measure as *proper* sequences, following the terminology of Zvonkin and Levin [4]). We focus in particular on the growth rates of functions of the form $f(X, n) = -\log \mu(X \upharpoonright n)$ for various computable measures μ and μ-random sequences X. As we demonstrate, these growth rates can vary widely, depending on the choice of the underlying measure μ.

In the first half of the paper, we focus on the relationship between a class of sequences known as *complex sequences* and those sequences that are random with respect to a computable, continuous measure. First studied systematically by Kjos-Hanssen et al. [1] (but also studied earlier by Kanovič [2]), complex sequences are those sequences whose initial segment complexities are bounded below by some computable function. We characterize the complex proper sequences as the sequences that are random with respect to some computable *continuous* measure. This is done by studying the "removability" of μ-atoms, that is, sequences X such that $\mu(\{X\}) > 0$. We show that if a sequence X is complex and random with respect to some computable measure μ, we can define a computable, continuous measure ν such that X is random with respect to ν by removing the μ-atoms that are in some sense near X. It is natural to ask whether this removal of atoms can always be carried out while preserving *all* non-atomic random sequences simultaneously, again assuming that all of these random sequences are complex. We show that this is not the case.

Using this characterization of complex sequences through computable continuous measures, we establish new results on the relationship between the notions of avoidability, hyperavoidability, semigenericity, and not being random for any computable, continuous measure. More specifically, when restricted to the collection of proper sequences, we show that these four notions are equivalent to being complex. We also study the granularity of a computable, continuous measure μ and show that the inverse of the granularity function provides a uniform lower bound for the initial segment complexity of μ-random sequences.

In the second half of the paper, we turn our attention to atomic computable measures, i.e., computable measures μ that have μ-atoms. First, we study atomic measures μ with the property that every μ-random sequence is either a μ-atom or is complex. We show that for such measures μ, even though the initial segment complexity of each non-atom μ-random sequence is bounded from below by some computable function, there is in general no uniform computable lower bound for every non-atom μ-random sequence. Next, we construct a computable atomic measure μ with the property that the initial segment complexity of each μ-random sequence dominates no computable function, and a computable atomic measure ν with the property that the initial segment complexity of each ν-random sequence is dominated by all computable functions. The former sequences are called *infinitely often anti-complex*, while the latter are known simply as *anti-complex*.

Lastly, we study two specific kinds of atomic measures: diminutive measures and trivial measures. Here, a measure μ is *trivial* if $\mu(\mathrm{Atoms}_\mu) = 1$, and diminutive measures are defined as follows.

Let $\mathcal{C} \subseteq 2^\omega$.

(i) $\mathcal{C}$ is *diminutive* if it does not contain a computably perfect subclass.

(ii) Let μ be a computable measure, and let $(\mathcal{U}_i)_{i\in\omega}$ be the universal μ-Martin-Löf test. Then we say that μ is *diminutive* if $\mathcal{U}_i^c$ is a diminutive Π^0_1 class for every i.

We show that while every computable trivial measure is diminutive, the converse does not hold. The proof of this last statement gives an alternative, priority-free proof of the following known result.

Corollary (Kautz [3]). *There is a computable, non-trivial measure μ such that there is no Δ^0_2, non-computable $X \in \mathrm{MLR}_\mu$.*

[1] Bjørn Kjos-Hanssen, Wolfgang Merkle, and Frank Stephan. Kolmogorov complexity and the recursion theorem. *Trans. Amer. Math. Soc.* **363** (2011), no. 10, 5465–5480.
[2] Max I. Kanovič. The complexity of the enumeration and solvability of predicates. *Dokl. Akad. Nauk SSSR* **190** (1070), 23–26.
[3] Steven M. Kautz. *Degrees of random sets.* PhD thesis, Cornell University, 1991.
[4] Alexander K. Zvonkin and Leonid A. Levin. The complexity of finite objects and the basing of the concepts of information and randomness on the theory of algorithms. *Uspehi Mat. Nauk* **25**(6(156)):85– 127, 1970.

Gregory Igusa

Generic computability and coarse computability

Joint work with Peter Cholak and Rod Downey.

Abstract: In 2012, Jockusch and Schupp introduced "generic computability" and "coarse computability" the recursion-theoretic content of algorithms that are only able to compute most of the bits of a real. A generic computation of a real is a computation that usually halts: a partial recursive function whose domain has density 1 (in the sense of asymptotic density on $\mathbb{N}$) such that the partial recursive function correctly computes the real on its domain. Similarly, a coarse computation is a computation that is usually correct: a total recursive function that is correct about the real on density 1.

To many people, at first glance it would appear that one of these two notions should imply the other. Perhaps a generically computable real should also be coarsely computable by an algorithm that guesses when it does not know the answer, or a coarsely computable real should also be generically computable by an algorithm that does not guess when it does not know the answer. Neither of these two implications is true, but from the work various people have done on these two notions, there is a large quantity of evidence suggesting that the first implication is "almost" true.

This "almost implication" can be witnessed by many standards of measurement: The Turing degrees that contain an element that is computable in one sense but not the other, the densities other than 1 at which a real can be generically or coarsely approximated, the Turing information coded in an arbitrary coarse (resp. generic) description of a generically (resp. coarsely) computable real, and an implication between two other analogously defined degree structures.

In recent work, many of these "almost implications" have been used to transport theorems back and forth between generic reducibility and coarse reducibility, producing a situation in which the distinction between defining reducibilities uniformly vs nonuniformly turns out to be more relevant than the distinction between generic and coarse computability.

Rutger Kuyper

Preservation of randomness and genericity under symmetric differences

Joint work with Joseph S. Miller.

Abstract: Given a class $\mathcal{U}$ of subsets of the natural numbers, let us say that a subset A of the natural numbers is $\mathcal{U}$-stabilising if the symmetric difference (or bitwise addition) $A + X$ of A and X is an element of $\mathcal{U}$ for every element X of $\mathcal{U}$.

Several people have asked, using varying terminology, to characterise the Martin-Löf-stabilising sets, i.e. the sets A such that $X + A$ is Martin-Löf random for every Martin-Löf random set X. We show that the Martin-Löf-stabilising sets are exactly the K-trivial sets.

We also show that the 1-generic-stabilising sets are exactly the computable sets, using an argument which (surprisingly) makes use of Kolmogorov complexity and facts about the K-trivial sets.

Wolfgang Merkle

Being low for K along sequences and elsewhere

Joint work with Liang Yu.

Abstract: Given a set D of strings, say a sequence X is low for prefix-free Kolmogorov complexity K on D in case access to X as an oracle does not improve the prefix-free complexity of any string in D by more than an additive constant. Furthermore, say X is weakly low for K on D in case X is low for K on some, then necessarily infinite, subset of D. The usual notions of being low for K and being weakly low for K are obtained by letting D be equal to the set of all strings. We investigate into the question what can be said about sequences that are low for K or weakly low for K on specific sets of strings, e.g., on the set of initial segments of some sequence. Accordingly, say X is low or weakly low for K along a sequence A in case X is low or weakly low, respectively, for K on the set of initial segments of A. More specifically, for an infinite subset I of the natural numbers, say X is low or weakly low for K along A on I in case X is low or weakly low, respectively, for K on the set of initial segments of A with length in I.

Among others, we demonstrate the following results. If a sequence X is not low for K, then for any sequence A and any infinite computable set I, the sequence X is not low along A on I. As an immediate corollary, a sequence X is low for K if and only if X is low along all sequences on all infinite computable sets if and only if X is low along some sequence on some infinite computable set. Furthermore, in case a sequence X is weakly low for K, then X is weakly low along almost all sequences (in the sense of Lebesgue measure). Hence X is weakly low for K if and only if X is weakly low for K along almost all sequences if and only if X is weakly low for K along some sequence. Finally, for any infinite set D of strings, the set of sequences X such that X is low for K on D has measure 0 whereas the set of sequences X such that X is weakly low for K on D has measure 1

Benoit Monin

A unifying approach to the Gamma question

Abstract: The Gamma question was formulated by Andrews et al. in "Asymptotic density, computable traceability and 1-randomness" (1). It is related to the recent notion of coarse computability which stems from complexity theory. The Gamma value of an oracle set measures to what extent each set computable with the oracle is approximable in the sense of density by a computable set. The closer to 1 this value is, the closer the oracle is to being computable. This notion has been extended in (1) to Turing degrees. The authors of (1) provided examples of degrees with a Gamma value of 0, some with a Gamma value of 1/2, and they showed that the computable degree is the only one with a Gamma value larger than 1/2 (its Gamma value being 1). We will present various results unifying the known examples of degrees with a Gamma value of 0 and 1/2, highlighting some new connections between these notions and algorithmic randomness.

Satyadev Nandakumar

Pseudorandomness and Dimension

Joint work with Manindra Agrawal, Diptarka Chakraborty and Debarati Das.

Abstract: In computational complexity theory, a probability distribution on a finite set of finite strings is said to be pseudorandom if no polynomial-size circuit is able to distinguish it from the uniform distribution on the same set. In this talk, we give an overview of a recent attempt to apply the theory of effective fractal dimension to define how close an arbitrary distribution is to being pseudorandom and relate it to known computational analogues of entropy. We also study the problem of whether it is possible to extract pseudorandom bits from a general distribution, analogous to the problem of extracting random bits from a pseudorandom source.

Jake Pardo

Gambling against some odds

Abstract: In their paper *Gambling Against All Odds*, Bavly and Peretz [1] compare the relative strengths of martingales that take their values from (potentially) different sets of real numbers, i.e. restricted value martingales. The area of restricted value martingales has been studied before; for example in Bienvenu et al. [2], Chalcraft, Dougherty, et al. [3], Peretz [4], and Teutsch [5]. A restricted value martingale is defined in the following way:

Definition. *Let $S \subseteq \mathbb{R}^+$; an S-martingale is any martingale M that satisfies*

$$|M(\sigma 1) - M(\sigma)| \in S$$

for all $\sigma \in 2^{<\mathbb{N}}$.

Recall that the *success set* of a martingale M is

$$succ(M) = \{X \in 2^{\mathbb{N}} : \lim_{n\to\infty} M(X \upharpoonright n) = \infty\}.$$

Then we can define the following:

Definition. *Let $A, B \subseteq \mathbb{R}^+$.*

- *B singly anticipates A if for every A-martingale X, there exists a B-martingale Y such that*
$$succ(X) \subseteq succ(Y).$$
Otherwise we say A singly evades B.

- *B (countably) anticipates A if for every A-martingale X, there exists a countable set of B-martingales $\{Y_1, Y_2, \dots\}$ such that*
$$succ(X) \subseteq \bigcup_{i\in\mathbb{N}} succ(Y_i).$$
Otherwise we say A (countably) evades B.

In their recent paper Bavly and Peretz [1] proved the following two results:

Theorem (Bavly/Peretz)**.** *Let $A, B \subseteq \mathbb{R}^+$ be such that $\sup A < \infty$ and $0 \notin \overline{B \setminus \{0\}}$; then B anticipates A if and only if A scales into B, i.e. there is an $r \in \mathbb{R}^+$ such that $rA \subseteq B$.*

Theorem (Bavly/Peretz)**.** *Let $A, B \subseteq \mathbb{R}^+$, and assume B is well-ordered; then B anticipates A if and only if A scales into B.*

Bavly and Peretz asked what can be shown in the case that 0 is an accumulation point of B. In this paper, the following result is shown.

Theorem. *Let $A = \mathbb{N}$ and $B = \{x^{-n} : n \in \mathbb{Z}\}$ for $x \in \mathbb{R}^+$ with $x > \frac{1+\sqrt{5}}{2}$; then A singly evades B.*

The proof is based on a betting game between two gamblers, gambler 1 using an A-martingale as a strategy and gambler 2 using a B-martingale, in which a strategy for the A-martingale gambler is produced. At the beginning of each stage of the strategy, if gambler 2 makes a relatively large bet, gambler 1 is able to punish their greedy play by taking both players down a punishing path of gambling, and if gambler 2 makes a relatively small bet, then gambler 1 is able to gain money at an appreciably faster rate than gambler 2 by taking both players along a route that rewards both gamblers (but gambler 1 moreso). This method shows some interesting connection to number theory, in particular, the n-Fibonacci numbers.

[1] Bavly and Peretz. How to gamble against all odds. *Games and Economic Behavior* 2014.
[2] Bienvenu, Stephan, and Teutsch. How Powerful Are Integer-Valued Martingales? *Theory of Computing Systems* **51** (2012), no. 3, 330–351.
[3] Chalcraft, Dougherty, Freiling, and Teutsch. How to Build a Probability-Free Casino. *Information and Computation* **211** (2012), 160–164.
[4] Peretz. Effective Martingales with Restricted Wagers. On arxiv 2013.
[5] Teutsch. A Savings Paradox for Integer-Valued Gambling Strategies. *International Journal of Game Theory* 2013.

Ron Peretz

Effective martingales with restricted wagers

Abstract: The classic model of computable randomness considers martingales that take real or rational values. Recent work shows that fundamental features of the classic model change when the martingales take integer values. We compare the prediction power of martingales whose wagers belong to three different subsets of rational numbers:

(a) all rational numbers,

(b) rational numbers excluding a punctured neighborhood of 0, and

(c) integers.

We also consider three different success criteria:

(i) accumulating an infinite amount of money,

(ii) consuming an infinite amount of money, and

(iii) making the accumulated capital oscillate.

The nine combinations of (a)–(c) and (i)–(iii) define nine variants of computable randomness. We provide a complete characterization of the relations between these notions, and show that they form five linearly ordered classes.

Cristóbal Rojas

Computability and complexity of random perturbations of dynamical systems

Joint work with M. Braverman and J. Schneider.

Abstract: A discrete-time dynamical system is specified by a function f from a space X to itself. One of the most important problems in the study of dynamical systems is to understand the limiting or asymptotic behavior of such systems; in particular, the limiting distribution of the sequence of iterates

$$x, f(x), f(f(x)), \ldots$$

Combinations of such distributions give rise to the *invariant measures* of the system, which describe the asymptotic behavior in statistical terms. In this talk we will discuss recent results on computability and complexity of such invariant measures. In particular, we will present tight bounds on the space-complexity of computing the ergodic measure of a low-dimensional discrete-time dynamical system, affected by Gaussian random perturbations.

Jeffrey O. Shallit

Separating Words Using Automata

Abstract: One of the simplest and most mysterious problems in automata theory is the problem of separating words: given two words x, w of length at most n, determine a good bound on the number of states needed in the smallest finite automaton accepting one of $\{x, w\}$ and rejecting the other. The upper and lower bounds currently known for this problem are widely separated. In this talk I will survey what is known about the problem.

Mariya Soskova

Definability and automorphisms in the enumeration degrees

Abstract: I will describe the connection between two problems in the structure of the enumeration degrees. The first one concerns the existence of nontrivial automorphisms of the structure. The second one is to understand what relations on degrees are first order definable in the language of the structure. I will outline what we know about definability in the enumeration degrees and finish with some open problems and possible future directions of research.

Kohtaro Tadaki

The Principle of Typicality

Abstract: This is a sequel to our presentation at CCR 2014. The notion of probability plays a crucial role in quantum mechanics. It appears as the Born rule. In modern mathematics which describes quantum mechanics, however, probability theory means nothing other than measure theory, and therefore any operational characterization of the notion of probability is still missing in quantum mechanics. At CCR 2014 we presented an alternative rule to the Born, using the notion of Martin-Löf randomness with respect to Bernoulli measure for specifying the property of the results of quantum measurements in an operational way. This alternative rule is about pure states. In this talk we generalize this alternative rule over mixed states. We then show that all of our new rules about quantum measurements based on algorithmic randomness, including the alternative rule to the Born rule about pure states and its generalization over mixed states, can be derived from a single postulate, called the principle of typicality, in a unified way.

Hayato Takahashi

An example of non-computable conditional probability for generalized Lambalgen's theorem

Abstract: Lambalgen's theorem (1987) [6] says that a pair of sequences $(x^\infty, y^\infty) \in \Omega^2$ is Martin-Löf (ML) random w.r.t. the product of uniform measures iff x^∞ is ML-random and y^∞ is ML-random relative to x^∞, where Ω is the set of infinite binary sequences. In [7, 4, 5], generalized Lambalgen's theorem is studied for computable correlated probabilities. We study generalization of Lambalgen's theorem without assuming computability of conditional probabilities using the notion of Hippocratic (blind) randomness [3, 2]. In [1], a counter-example for the generalization of Lambalgen's theorem is shown when the conditional probability is not computable. We show an example of conditional probability (Theorem) that is not computable for all random parameters but generalization of Lambalgen's theorem still holds for all random parameters.

In [4], the section of random set on product space is defined as random sequences with respect to conditional probability. In this talk I would like to talk about the validity of the definition from the Bayesian statistical point of view.

Let S be the set of finite binary strings and $\Delta(s) := \{sx^\infty | x^\infty \in \Omega\}$ for $s \in S$, where sx^∞ is the concatenation of s and x^∞. Let $X = Y = \Omega$ and P be a computable probability on $X \times Y$. P_X and P_Y are marginal distribution on X and Y, respectively. In the following we write $P(x, y) := P(\Delta(x) \times \Delta(y))$ and $P(x|y) := P(\Delta(x)|\Delta(y))$ for $x, y \in S$.

Let $\mathcal{R}^P$ be the set of ML-random points and $\mathcal{R}^P_{y^\infty} := \{x^\infty \mid (x^\infty, y^\infty) \in \mathcal{R}^P\}$. In [4], it is shown that conditional probabilities exist for all random parameters, i.e.,

$$\forall x \in S,\ y^\infty \in \mathcal{R}^{P_Y} \quad P(x|y^\infty) := \lim_{y \to y^\infty} P(x|y) \text{ (the right-hand-side exist)}$$

and $P(\cdot|y^\infty)$ is a probability on $(\Omega, \mathcal{B})$ for each $y^\infty \in \mathcal{R}^{P_Y}$.

Let $\mathcal{H}^{P(\cdot|y^\infty), y^\infty}$ be the set of Hippocratic random sequences w.r.t. $P(\cdot|y^\infty)$ with oracle y^∞.

Theorem ([4, 5])**.** *Let P be a computable probability on $X \times Y$. Then*

$$\mathcal{R}^P_{y^\infty} \supseteq \mathcal{H}^{P(\cdot|y^\infty), y^\infty} \text{ for all } y^\infty \in \mathcal{R}^{P_Y}. \tag{1}$$

The equality holds if the conditional probability is computable with oracle y^∞.

The next theorem shows an example that equality does not hold in (1).

Theorem (Bauwens [1])**.** *There is a computable probability P on $X \times Y$ and $y^\infty \in \mathcal{R}^{P_Y}$ such that*
(a) $P(\cdot|y^\infty)$ is not computable and (b) $\mathcal{R}^P_{y^\infty} \neq \mathcal{H}^{P(\cdot|y^\infty), y^\infty}$.

The following is an example that equality holds in (1) even if the conditional probability is not computable for all random parameters.

Theorem. *There is a computable probability P on $X \times Y$ such that for all $y^\infty \in \mathcal{R}^{P_Y}$,*
(a) $P(\cdot|y^\infty)$ is not computable and (b) $\mathcal{R}^P_{y^\infty} = \mathcal{H}^{P(\cdot|y^\infty), y^\infty}$.

References

[1] B. Bauwens, 2014. preprint.

[2] L. Bienvenu, P. Gacs, M. Hoyrup, C. Rojas, and A. Shen. Algorithmic tests and randomness with respect to a class of measures, 2011. arxiv:1103.1529v2.

[3] Bjørn Kjos Hanssen. The probability distribution as a computational resource for randomness testing. *Journal of Logic and Analysis*, 2(10):1–13, 2010.

[4] H. Takahashi. On a definition of random sequences with respect to conditional probability. *Inform. and Compt.*, 206:1375–1382, 2008.

[5] H. Takahashi. Algorithmic randomness and monotone complexity on product space. *Inform. and Compt.*, 209:183–197, 2011.

[6] M. van Lambalgen. *Random sequences.* PhD thesis, Universiteit van Amsterdam, 1987.

[7] V. G. Vovk and V. V. V'yugin. On the empirical validity of the Bayesian method. *J. R. Stat. Soc. B*, 55(1):253–266, 1993.

Dan Turetsky

The complexity of free abelian groups

Abstract: Vector spaces are defined by a set of first-order axioms; the theorem "every vector space has a basis" is second-order, but is a consequence of these axioms. In contrast, free abelian groups are typically defined by having a basis. It is then natural to wonder if free abelian groups could also be defined by some set of first-order axioms. A negative answer might take the form of a theorem that the index set of computable free abelian groups is not arithmetic. Unfortunately, this theorem is false, at least in standard computability theory. It can be rescued, however, by moving to uncountable computability theory.

Linda Westrick

Seas of squares with sizes from a co-enumerable set

Abstract: Three prominent classes of two-dimensional subshifts are the *shifts of finite type* (SFTs), the *sofic shifts*, and the *effectively closed shifts.* SFTs are obtained by forbidding finitely many local patterns. A shift is sofic if it is a *factor* of a SFT, meaning that it is the image of a SFT under a continuous shift-invariant map. And a shift is effectively closed if it is obtained by forbidding a c.e. set of local patterns.

In general, every SFT is sofic and every sofic shift is effectively closed, but the reverse implications do not hold. We would like to know what conditions on a c.e. set of forbidden patterns would guarantee that the resulting shift is not just effectively closed, but sofic. The primary purpose of this work is to expand our inventory of examples and techniques related to this question.

For each co-enumerable $S \subseteq \mathbb{N}$, consider a two-dimensional subshift X_S on the alphabet $\{\text{black}, \text{white}\}$ whose elements consist of black squares of various sizes on a white background, where the side length of each square is in S. We show that each X_S is sofic.

The entropy of a sofic shift is always less than or equal to the entropy of the SFT of which is is a factor. In two and more dimensions, it is an open question whether every sofic shift is the factor of a SFT of the same entropy. Our seas of squares do not provide a counterexample: we show that each X_S is a factor of a SFT of the same entropy.

To prove our results, we build upon the self-similar Turing machine tiling construction of Durand, Romashchenko and Shen (2010). They showed that the following class of effectively closed shifts are sofic (this result was also obtained independently by Aubrun and Sablik). For each one-dimensional effectively closed subshift W, consider a two-dimensional shift X_W whose elements consist of vertical stripes arranged so that the common row is an element of W. In other words, X_W is obtained by "stretching" W into two dimensions.

In their construction, arrays of Turing machines operating a multiple scales work together to "read" the common row, compute the forbidden words of W, and kill any elements whose common row contains a forbidden word. The "reading" process makes strong use of the fact that each bit of the common row is repeated infinitely often along an entire column, so the machines have many opportunities to work together to access it.

In our construction, a similar array of Turing machines "measures" the sizes of the squares in a potential element of X_S, computes the forbidden sizes, and kills the element if it sees a square of a forbidden size. However, an extra challenge is that each machine must measure and store the size of every single square in its vicinity, since sizes are not necessarily repeated and thus no machine can compensate for another machine overlooking a square. A high density of information must also be communicated between the machines, creating additional challenges.

General Information

VENUE The conference is held in the Physical Science Building on the Mānoa campus. All lectures are in the auditorium PSB 217 which is reserved from 8am to 4:30pm, Monday through Friday.

PROGRAM COMMITTEE Laurent Bienvenu (Paris, France), Rod Downey (Wellington, New Zealand) (co-chair), Johanna Franklin (Hempstead, New York, USA), Denis Hirschfeldt (Chicago, Illinois, USA) (co-chair), Bjørn Kjos-Hanssen (Honolulu, Hawaii, USA), Jack Lutz (Ames, Iowa, USA), Elvira Mayordomo (Zaragoza, Spain), Joe Miller (Madison, Wisconsin, USA), Kenshi Miyabe (Tokyo, Japan), Andrei Romashchenko (Montpellier, France), Henry Towsner (Philadelphia, Pennsylvania, USA), Nikolai Vereshchagin (Moscow, Russia).

LOCAL ORGANIZATION AND SPONSORS The auditorium fees and coffee service are sponsored by the Mathematics Department at University of Hawaiʻi at Mānoa. Financial support is given by the National Science Foundation (NSF) under Grant No. 1545707 and by the Simons Foundation under a Collaboration Grant for Mathematicians (#315188 to Bjørn Kjos-Hanssen) . The local organizing committee consists of Bjørn Kjos-Hanssen (chair and PI), Mushfeq Khan (co-PI), Achilles Beros, Dale Myers, David Ross, and Dusko Pavlovic.

Department of Mathematics
2565 McCarthy Mall
Honolulu, HI 96822

E-Mail: bjoern.kjos-hanssen@hawaii.edu