Obligations in a context

Simplified semantics for the deontic logic of contrary-to-duties

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Neither Reagan nor Gorbachev must be told the secret.

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• But if one of them is told the secret, then the other one should be told as well. A *contrary-to-duty obligation*. 
CTD obligations do not formalize well if $O(B | A)$ is taken as $A \rightarrow O(B)$ or $O(A \rightarrow B)$. A separate dyadic operator $O(B | A)$ is needed.

Carmo and Jones (1997, 2002, 2013) base it on a function

$$\text{ob} : 2^W \rightarrow 2^{2^W}$$

with

$$\models_\alpha O(A \mid B) \iff \| A \| \in \text{ob}(\| B \|),$$

where $W$ is the set of worlds.
Here we propose a simpler function $F : 2^W \rightarrow 2^W$ and

$$\text{ob}(X) = \{Y : Y \supseteq F(X)\}.$$ 

Because of the idea of a given context as a set of worlds, it seems we cannot further simplify to just a relation on worlds $R \subseteq W \times W$. An alternative notion is obtained by

$$Y \in \text{ob}(X) \iff Y \cap X = F(X).$$
5(a) \( \text{ob}(X) \neq \emptyset \).

5(b) If \( Y \cap X = Z \cap X \) then \( Y \in \text{ob}(X) \) iff \( Z \in \text{ob}(X) \).

5(c) If \( Y \in \text{ob}(X) \) and \( Z \in \text{ob}(X) \) then \( Y \cap Z \in \text{ob}(X) \).

5(d) If \( Y \subseteq X \) and \( Y \in \text{ob}(X) \) and \( X \subseteq Z \), then \( (Z \setminus X) \cup Y \in \text{ob}(Z) \).

5(e) If \( Y \subseteq X \) and \( Z \in \text{ob}(X) \) and \( Y \cap Z \neq \emptyset \), then \( Z \in \text{ob}(Y) \).
Carmo and Jones’ conditions on $\text{ob}$

5(a) $\text{ob}(X) \neq \emptyset$. 
5(a) \( \text{ob}(X) \neq \emptyset \). We only require this for \( X \neq \emptyset \). In any case \( \text{ob}(\emptyset) \) is not that interesting.
5(b) If $Y \cap X = Z \cap X$ then $Y \in \text{ob}(X)$ iff $Z \in \text{ob}(X)$. 

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5(c) If $Y \in \text{ob}(X)$ and $Z \in \text{ob}(X)$ then $Y \cap Z \in \text{ob}(X)$. 
5(c) If $Y \in \text{ob}(X)$ and $Z \in \text{ob}(X)$ then $Y \cap Z \in \text{ob}(X)$. Follows from $\text{ob}(X) = \{Y : Y \supseteq F(X)\}$. 
5(d) If $Y \subseteq X$ and $Y \in \text{ob}(X)$ and $X \subseteq Z$, then $(Z \setminus X) \cup Y \in \text{ob}(Z)$. 

This becomes the condition

$F(X \cap Y) \supseteq F(X) \cap Y$:

our standards of perfection can only be relaxed, not strengthened, when restricting the context. If we take $Y \in \text{ob}(X)$ to mean $Y \cap X = F(X)$ then 5(d) is just wrong.
5(d) If $Y \subseteq X$ and $Y \in \text{ob}(X)$ and $X \subseteq Z$, then $(Z \setminus X) \cup Y \in \text{ob}(Z)$. This becomes the condition $F(X \cap Y) \supseteq F(X) \cap Y$: our standards of perfection can only be relaxed, not strengthened, when restricting the context. If we take $Y \in \text{ob}(X)$ to mean $Y \cap X = F(X)$ then 5(d) is just wrong.
5(e) If $Y \subseteq X$ and $Z \in \text{ob}(X)$ and $Y \cap Z \neq \emptyset$, then $Z \in \text{ob}(Y)$. 

Must be weakened. We use: $F(X \cap Y) = F(X) \cap Y$ whenever $F(X) \cap Y \neq \emptyset$: standards of perfection should only be relaxed when absolutely necessary. However, when $Z \in \text{ob}(X)$ is taken as not $Z \supseteq F(X)$ but $Z \cap X = F(X)$, so $Z$ consists of only ideal worlds, then it is okay.
5(e) If $Y \subseteq X$ and $Z \in \text{ob}(X)$ and $Y \cap Z \neq \emptyset$, then $Z \in \text{ob}(Y)$. Must be weakened. We use: $F(X \cap Y) = F(X) \cap Y$ whenever $F(X) \cap Y \neq \emptyset$: standards of perfection should only be relaxed when absolutely necessary. However, when $Z \in \text{ob}(X)$ is taken as not $Z \supseteq F(X)$ but $Z \cap X = F(X)$, so $Z$ consists of only ideal worlds, then it is okay.
Suppose $W = \{a, b, c, d, e\}$ where $a$ is mandatory:

- $\{a\} \in \text{ob}(W)$.
- $\{a, b, c\} \in \text{ob}(W)$ by 5(d) since it is “if $\{a, d, e\}$ then $\{a\}$” (which is equivalent to “not $d$ or $e$”). And then
- $\{a, b, c\} \in \text{ob}(\{b, c, d, e\})$ by 5(e). Similarly,
- $\{a, b, d\} \in \text{ob}(\{b, c, d, e\})$. So
- $\{b, c\} \in \text{ob}(\{b, c, d, e\})$ and $\{b, d\} \in \text{ob}(\{b, c, d, e\})$ by 5(b).

CJ (2002) defined by 5(c$^-$):

If $Y \in \text{ob}(X)$ and $Z \in \text{ob}(X)$ and $X \cap Y \cap Z \neq \emptyset$, then $Y \cap Z \in \text{ob}(X)$.

By 5(c$^-$), $\{b\} \in \text{ob}(\{b, c, d, e\})$. But this was obtained without using any desirability property of $b$. 
More generally, in “general position”, if

- \( \|A\| \in \text{ob}(W) \), then
- \( \|A\| \in \text{ob}(\|A \lor B\|) \) by 5(e). Then
- \( \|A \lor \neg(A \lor B)\| = \|A \lor \neg B\| \in \text{ob}(W) \) by 5(d) (slide 6). Then
- \( \|A \lor \neg B\| \in \text{ob}(\|\neg A\|) \) by 5(e). So
- \( \|\neg B\| \in \text{ob}(\|\neg A\|) \) by 5(b).

But this is absurd, as \( B \) was fairly arbitrary.
Prisoner’s dilemma

\( D_i = \text{player } i \text{ defects.} \)
We adopt player 1’s point of view.

\[
O(D_1 \land \neg D_2)
\]

\[
O(\neg D_1 \mid D_1 \leftrightarrow D_2).
\]

This is another way to try to defeat condition 5(e).
CJ might argue that \( D_1 \land \neg D_2 \) is the “true” obligation, whereas \( D_1 \) and \( \neg D_2 \) are not.
They might argue that the true obligation is to minimize your own number of years in jail. Then, by first-order logic, it follows that \( D_1 \land \neg D_2 \) should hold.
Let \( \text{ob}(X) \) consist of all sets containing the most favorable element of \( X \). That is, we have a valuation \( v : W \to \mathbb{N} \) on worlds and we let

\[
F(X) = \{ u \in X : (\forall x \in X)(v(x) \leq v(u)) \}.
\]

We can recover an ordering by

\[
a \leq b \iff (\forall X)(a \in F_X \to b \in F_X).
\]
A weak version of 5(e), 5(e⁻):
If \( Y \subseteq X \) and \( Z \in \operatorname{ob}(X) \) and \( Y \cap Z \neq \emptyset \), and \( \overline{Z} \notin \operatorname{ob}(Y) \), then \( Z \in \operatorname{ob}(Y) \).

will help if the worlds are (strictly) linearly ordered in value.

Let us consider a model in which they are not. Let’s say we want to maximize our profits and minimize the variance in our profits. Then some pairs \( (\mu_1, \sigma_1^2) \) are better than others \( (\mu_2, \sigma_2^2) \) but for some pairs it is hard to declare a preference. So let’s say world \( a \) is best overall, \( b \) and \( c \) are incomparable, and \( d \) is worst. Then

- \( \{a\} \in \operatorname{ob}(W) \), so
- \( \{a\} \in \operatorname{ob}(\{a, b\}) \) by 5(e⁻), so
- \( \{a, c, d\} \in \operatorname{ob}(W) \) by 5(d), so
- \( \{a, c, d\} \in \operatorname{ob}(\{b, c, d\}) \) by 5(e⁻), since \( \{a, c, d\} \) is still possible and in fact not forbidden in this new context.

And so we have a preference for \( c \) over \( b \), which is bad.
History of condition 5(e)

- Proposed by Carmo and Jones (published 1997) with the motivation $Y = \operatorname{av}(w)$, the set of actually possible versions of $w$, and $X = \operatorname{pv}(X)$, the set of potentially or ideally possible versions of $X$. 

- K-H gives a counterargument (term paper, 1996)

- Carmo and Jones discuss the counterargument and defeat it by weakening condition 5(c) (Deontic logic and contrary-to-duties, 2002)

- Completeness results published (2013)

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Fence scenario

• There ought to be no dog.
• If there is dog, there ought to be a frontyard fence and a backyard fence.
• No point in having a frontyard fence if there is no backyard fence.

may appear to defeat 5(e): let $Y$ be “there is no frontyard fence” and let $Z$ be “there is a backyard fence”, and let $X = W$ (no restriction).

Fix Use 5(e−): require $W \setminus Y \not\in \text{ob}(X)$ in the antecedent.

CJ No fix is needed! “There ought to be a backyard fence” is not a true obligation, only the conjunction of backyard and frontyard is.
Alice should drive on the right-hand side.

Bob should drive on the right-hand side.

Carly should drive on the right-hand side.

Dave should drive on the right-hand side.

Eve should drive on the right-hand side.
Traffic scenario

Two cars are driving down the same street in opposite directions.

• $A$ is the proposition that car $A$ is driving on the right side of the street.
• $B$ is the proposition that car $B$ is driving on the right side of the street.

1 There is a primary obligation that $A \leftrightarrow B$. This one is implied by the laws of all countries.

2 Then there is a secondary obligation (which is easier to administrate, and which implies the primary one) that $A \land B$. However, some countries instead use $\neg A \land \neg B$.

Note however that because of the primary obligation, if $\neg A$ is a fixed fact then $\neg B$ becomes an obligation.
We may note that our new $O(\cdot | \cdot)$ definition and semantics makes it a normal conditional logic (i.e., extending $CK$), and its models are standard conditional models, in the sense of Chellas 1980, if we define $A \Rightarrow B$ as $O(B | A)$. (In fact they fit a special case of standard conditional models in which the truth of $A \Rightarrow B$ does not depend on the current world.) Chellas does not consider this option. He considers to define $O(B | A)$ as $A \Rightarrow O(B)$, and he considers using minimal models (like CJ do) rather than the simpler standard models.
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