**Theorem.** Changing the value \( f(x) \) changes the vertical position of the graph. Changing the argument \( x \) changes the horizontal position in the opposite direction.

<table>
<thead>
<tr>
<th>( f(x) + a )</th>
<th>( f(x) - a )</th>
<th>( f(x) + d )</th>
<th>( f(x) - d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) ) up 1</td>
<td>( f(x) ) down 1</td>
<td>( f(x) ) right 1</td>
<td>( f(x) ) left 1</td>
</tr>
</tbody>
</table>

**Basic graphs**

- **Graph Reflect in y-axis.**
- **Graph Reflect in x-axis.**
- **Graph Shift left 1.**
- **Graph Shift right 1.**

- **Graph Parabola with roots 0, 1.**
- **Graph Vertical shift 1.**

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( f(x) + a )</th>
<th>( f(x) - a )</th>
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<td>( f(x) ) + 1</td>
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<td>( f(x) ) - 1</td>
</tr>
</tbody>
</table>

**Key transformations:**
- Horizontal shift: \( f(x+c) \)
- Vertical shift: \( f(x)+d \)
- Reflection in x-axis: \( f(-x) \)
- Reflection in y-axis: \( -f(x) \)
- Vertical stretch: \( af(x) \)
- Horizontal stretch: \( f(bx) \)
- Vertical reflection: \( f(x)+1 \)
- Horizontal reflection: \( f(x+1) \)

By changing the value \( f(x) \), the graph's vertical position is altered. By changing the argument \( x \), the horizontal position is changed in the opposite direction.

For a formula with several shifts and reflections of \( f(x) \), rewrite it in the graph-translation form:

\[ f(x) = af(bx - c) + d \]

The shifts and reflections occur in the left-to-right order.

- A negative \( c \) gives a horizontal reflection. The horizontal shift is determined by \(|c| \); right if \( c \) is positive, left if \( c \) is negative.
- A positive \( d \) gives a vertical reflection. The vertical shift is determined by \( d \); up if \( d \) is positive, down if \( d \) is negative.

- **Horizontal moves with argument changes:**
  - By \( x \rightarrow x+c \) the graph \( c \) units to the left.
  - By \( x \rightarrow x/b \) the graph stretches horizontally by a factor of \( b \).

- **Vertical moves with value changes:**
  - By \( f(x) \rightarrow f(x)+a \) the graph shifts \( a \) units up.
  - By \( f(x) \rightarrow f(x)-a \) the graph shifts \( a \) units down.

The value \( f(x) = \) the height; \( x \) is the horizontal position of a point on the graph. Changing \( f(x) \) changes the vertical position of the graph. Changing \( x \) changes the horizontal position of the coordinate system. Replacing \( x \) by \( x-2 \) shifts the coordinate system 2 units to the left.

Given \( f(x) = \sqrt{x} \), graph \( f(x+2), f(x-2), f(x^2), f(x^2) \).