Basic graphs  Know these graphs. Quiz on next page.
For each graph pick the correct function.
Theorem. For the graph of any function $f$,

vertical changes $\leftrightarrow$ changes in the value $f(x)$,

horizontal changes $\leftrightarrow$ changes in the argument $x$

and work in the opposite direction.

<table>
<thead>
<tr>
<th>$\uparrow$ up 1 unit</th>
<th>$\downarrow$ down 1</th>
<th>reflect vertically around x-axis</th>
<th>$\leftarrow$ left 1</th>
<th>$\rightarrow$ right 1</th>
<th>reflect horizontally around y-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)+1$</td>
<td>$f(x)-1$</td>
<td>$-f(x)$</td>
<td>$f(x+1)$</td>
<td>$f(x-1)$</td>
<td>$f(-x)$</td>
</tr>
</tbody>
</table>

* Horizontal changes are the opposite of what one would expect.

Horizontal changes come from replacing $x$.
Replacements work in opposite the expected direction.

In $y=f(x)$ replacing $y$ by $y-1$ gives $y-1=f(x)$ which is the same as $y=f(x)+1$ which shifts up one unit.
Vertical Moves

- Given $f(x) = |x|$

\[
f(x) = |x|, \quad f(x) + 2 = |x| + 2, \quad f(x) - 2 = |x| - 2, \quad -f(x) = -|x|
\]

The value $f(x) = \text{the height} = \text{the vertical position of a point on the graph.}$

Changing $f(x)$ changes the \textbf{vertical position of the graph}.

Adding 2 raises the graph 2 units.

Negating $f(x)$ reflects the graph vertically across the $x$-axis.
Horizontal Moves

Replacing $x$ by $x+2$ shifts the graph left 2 units.
Replacing $x$ by $x-2$ shifts the graph right 2 units.
Replacing $x$ by $-x$ reflects the graph horizontally across the $y$-axis.

$f(x)=\sqrt{x}$, $f(x+2)=\sqrt{x+2}$, $f(x-2)=\sqrt{x-2}$, $f(-x)$
Given \( f(x) \), find the graph of ---

- reflect in y-axis
- reflect in x and y-axis
- reflect in x-axis
- find the graph of \(-f(x)\).
- find the graph of \(f(-x)\).
- find the graph of \(f(x - 2)\).
Given \( f(x) \), find the functions for the other graphs.

- (1) is the graph of
- (2) is the graph of
\[ f(x) = \frac{1}{x}. \] Graph \( f \) and describe the shifts.

\[ \frac{1}{x} \] is a hyperbola

\[ \frac{1}{x} + 1 \] Shift up 1

\[ -\frac{1}{x} \] Reflect vertically in \( x \)-axis

\[ \frac{1}{x-1} \] Shift right 1

\[ \frac{1}{x+1} \] Shift left 1

\[ \frac{1}{-x} \] Reflect horizontally across the \( y \)-axis
Given a function $f$, to handle a formula with a sequence of shifts and reflections, rewrite it in the form

$$af(b(x - c)) + d$$

Then the shifts and reflections occur in the following order:
- A negative $a$ gives a vertical reflection.
- A negative $b$ gives a horizontal reflection.
- The horizontal shift is determined by the $c$. A positive $c$ is a right shift, a negative $c$ is a left shift.
- The vertical shift is determined by $d$. A positive $d$ is an upward shift, a negative $d$ is a downward shift.

Rewrite $3 - f(-2 - x)$ in the above form and then list the sequence of shifts and reflections.

$$-3 - f(-2 - x)$$
$$= -f(-x - 2) - 3$$
$$= -f(-(x + 2)) - 3$$
$$= -f(-(x - (-2))) + (-3)$$

The sequence of shifts and reflections is:
- vertical reflection around $x$-axis,
- horizontal reflection around the $y$-axis,
- horizontal shift left 2,
- vertical shift down 3.
Rewrite form: $af(b(x-c)) + d$

$f(x) = \frac{1}{x}$. Find the sequence of shifts and/or reflections which carry $f(x)$ to $f(-x-1)$.

- $f(-x-1) = f(-(x+1))$
- $f(1-x)$
Rewrite form: \( af(b(x - c)) + d \)

Find the sequence of shifts and/or reflections.

- \( \frac{1}{x+1} + 1 = f(x + 1) + 1 \)

Don’t just memorize steps, try to understand the math. If you understand, every test problem should be recognizable as equivalent to some problem you have done before. If don’t understand, there will be problems on the exam which you won’t recognize.
The graphs of $f$ and $g$ are

First rewrite it in the form $af(b(x-c)) + d$

- Graph $1 - g(1 - x)$
- Graph $2 - f(2 - x)$
Graph $2 - f(2 - x)$

$-f(-(x - 2)) + 2$
Graph $1 - g(1 - x)$ → Your problem.