Don’t just memorize steps, try to understand the math. If you understand, every test problem should be recognizable as equivalent to some earlier problem. If don’t understand, there will be problems on the exam which you won’t recognize.

**Definition.**

A *quadratic function* is a polynomial of degree 2:

\[ y = ax^2 + bx + c , \quad a \neq 0. \]

The graph is a parabola. Its *shape* is determined by \( a \). \( a \) determines the direction taken when moving away from the vertex (nose).

- If \( a > 0 \), the horns point up.
- If \( a < 0 \), the horns point down.
- If \( -1 < a < 1 \), the parabola is wider than \( y = x^2 \).
- If \( a < -1 \) or \( 1 < a \), then it is narrower than \( y = x^2 \).
For each equation, find the shape of the graph.

- \( y = \frac{1}{2}(x - 2)^2 \)
- \( y = \frac{1}{2}x - 2x^2 \)

(A) Narrower than \( x^2 \).

(B) Wider than \( x^2 \).

(C) Narrower than \( x^2 \).

(D) Wider than \( x^2 \).

(E) None of these
Given $y = ax^2 + bx + c$

Get the roots by factoring or using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$ No roots if $b^2 - 4ac < 0$.

**Completing the Square Theorem.** Every quadratic function may be written in the *completed square* form:

$$y = a(x - x_0)^2 + y_0$$

where $(x_0, y_0)$ is the *vertex* (or nose) of the parabola.

Given $ax^2 + bx + c$.

- Factor the $a$ out of the $ax^2 + bx$ part.
- Divide the new coefficient of $x$ by 2 and square. Add this to complete the square.
- Anything which is added must also be subtracted to preserve equality.
• Find the roots (the roots are the $x$-intercepts).

• Write in completed square form: $y = a(x - x_0)^2 + y_0$
  Note: $-$ after $x$, $+$ after the square $)^2$

• Graph. List both coordinates of the vertex on the graph.

- $y = 2(x - 1)^2$
- $y = -\frac{1}{2}(x + 1)^2$

- Find the vertex.

- Draw the graph.
Find the roots.

\[ y = 2x^2 - 2x \]

\[ y = 2x^2 + 4x \]
Write in completed square form: \( y = a(x - x_0)^2 + y_0 \).

- \( y = 2x^2 - 2x \)

- \( y = 2x^2 + 4x \)

- Find the vertex.

Find the vertex.

Draw the graph.

- \( y = x^2 + 2x - 1 \)

Vertex = ... (1/2, -1/2)
For Exam 2, know the area and volume formulas for triangles, rectangles, circles, boxes, cans (including curved surface area, this is area of the side of the can).

**WORD PROBLEMS**

• *Draw* the picture. Indicate the variables in the picture.

• *Write the given* equations which relate the variables.

• *Solve for the wanted* quantities.
The perimeter of a rectangle is 10 feet. Express the area $A$ in terms of the width $x$.

Given:

List the given.

Write the perimeter as a function of $x$.

The area of a rectangle is 10 square feet. Express the perimeter $P$ in terms of the width $x$.

Want:
The corner of a triangle lies on the line $y = 4 - x$.

Write the triangle’s area and perimeter in terms of the base $x$. 

$A = \text{area}$

$P = \text{perimeter}$
The **curved surface area** $S$ is the area of the can’s side, excluding the top and bottom.

$$S = 2\pi rh$$

The volume is:

$$V = \pi r^2 h$$
The height of a can (right circular cylinder) is four times the radius. Express the curved surface area $S$ as a function of the radius $r$. Given:

$S =$

The height of a can is three times the radius. Express the radius $r$ as a function of the volume $V$.Want:

$\sqrt[3]{V/3\pi}$
The area of an isosceles triangle is 16.

- Write the width $w$ in terms of the height $h$.

- Write the height $h$ in terms of its width $w$.

Given:

Want: