Math 140    Lecture 19

**Pythagorean Identities.**

\[
\begin{align*}
\sin^2 t + \cos^2 t &= 1 \\
\tan^2 t + 1 &= \sec^2 t \\
\cot^2 t + 1 &= \csc^2 t
\end{align*}
\]

- Simplify \( \csc t + \csc t \cot^2 t \)
  \[ \frac{\sec^2 t - \tan^2 t}{\sec^2 t - \tan^2 t} \]

- Simplify \( \sec t + \sec t \tan^2 t \)
  \[ \frac{\tan^2 t - \sec^2 t}{\tan^2 t - \sec^2 t} \]
At $x=0$, the line $y=x$ is tangent to the graph of $\sin(x)$. 
Graph of $\cos x$
Remember these basic shapes.

$\sin(x)$

$\cos(x)$

$-\sin(x)$

$-\cos(x)$
**Definition.** The *amplitude* of a function $f$ is half the difference between the max and min values of $f$.

- Find the amplitude $A$ and period $p$ of $f$, $g$.
- Find the amplitude $A$ and period $p$ of $h$.

![Graphs of functions $f$, $g$, and $h$ with points marked and amplitude and period values labeled.]

- $f$: $A = 3$, $p = 4$
- $g$: $A = 1$, $p = 8$
- $h$: $A = $, $p = $
Graph $y = 2\sin(x)$ over one period.

Graph $y = \sin(2x)$ over one period.

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**Theorem**

For $y = \pm A\sin(Bx)$ &

$y = \pm A\cos(Bx)$, $A, B>0$

amplitude: $A$

period: $p = \frac{2\pi}{B}$

compression factor: $B = \frac{2\pi}{p}$

The theorem requires a positive $B$. Take care of any negation sign in the argument, before applying the theorem.

To graph a trig function with amplitude $A$ and period $p$:

- Draw a box with length $p$
  and height from $-A$ to $A$.

- Divide it horizontally into four parts (one for each quadrant).

**Theorem** For $y = \pm A\tan(Bx)$, $B>0$

period: $p = \frac{\pi}{B}$

compression factor: $B = \frac{\pi}{p}$
**Theorem.** For $y = \pm A \sin(Bx)$ & $y = \pm A \cos(Bx)$, $A, B > 0$

- amplitude: $A$ ← always positive
- compression factor: $B$ ← always positive
- period: $p = 2\pi/B$ ← always positive

Graph over one period

- $y = 2\sin(2\pi x)$
  Amplitude $A = 2$
  Shape $= \sin = A$

- Compression factor $B =$

- Period $p =$

- $y = -3\cos(-\pi x)$ ... $p = 2$
**Theorem.** For \( y = \pm A \sin(Bx) \) & \( y = \pm A \cos(Bx), \ A, B > 0 \)

- amplitude: \( A \) ← always positive
- compression factor: \( B \) ← always positive
- period: \( p = 2\pi/B \) ← always positive

Graph over one period

- \( y = -3\cos(2x) \)
- Amplitude \( A = \)

- Shape =

- Compression factor \( B = 2 \)

- Period \( p = \)

\[ y = 2\sin(3x) \quad \ldots \quad p = 2\pi/3 \]
Basic shapes: \( y = \pm A \sin(Bx), \ y = \pm A \cos(Bx). \)

- \( \sin(x) \)
- \( -\sin(x) \)
- \( \cos(x) \)
- \( -\cos(x) \)

Find the equation for the graph. Write in the form \( y = \pm A \sin(Bx) \) or \( y = \pm A \cos(Bx). \)  

... \(-3 \cos\left(\frac{\pi}{2}x\right)\)

Amplitude:

Basic shape:

Period:

Compression factor:

Equation for the graph:
Find the equation for the graph. Write in the form $y = \pm A\sin(Bx)$ or $y = \pm A\cos(Bx)$. 

Amplitude: 

Basic shape: 

Period: 

Compression factor: 

Equation for the graph:

... $-2\sin\left(\frac{\pi}{4} x\right)$
- Amplitude: \( A = \)
- Basic shape:
- Period: \( p = \)
- Compression factor: \( B = \)
- Equation for the graph: \( y = \)
- **Amplitude**: $A =$
- **Basic shape**: 
- **Period**: $p =$
- **Compression factor**: $B =$
- **Equation for the graph**: $y =$
**Recall.** For $C > 0$, the graph of $f(x - C)$ is $f(x)$ shifted $C$ units to the right.

The *phase shift* is the amount $C$ subtracted from $x$.

<table>
<thead>
<tr>
<th>Function</th>
<th>Phase shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x - C)$</td>
<td>$C$</td>
</tr>
<tr>
<td>$f(x + C) = f(x - (-C))$</td>
<td>$-C$</td>
</tr>
<tr>
<td>$f(2x - C) = f(2(x - \frac{C}{2}))$</td>
<td>$\frac{C}{2}$</td>
</tr>
<tr>
<td>$f(\pi x + 2) = f(\pi(x + \frac{2}{\pi})) = f(\pi(x - (-\frac{2}{\pi})))$</td>
<td>$-\frac{2}{\pi}$</td>
</tr>
</tbody>
</table>

Inside the argument of $f$, factor out the coefficient of $x$.

Find the phase shift $C$.

- $\cos(\frac{1}{2}x + 2\pi) \ldots -4\pi$
- $\sin(2x - \pi)$
Find the phase shift $C$.

- $-2 \cos(2\pi x + \pi/2)$

- $-\sin(\frac{\pi}{2}x - \pi)$ ... $C = 2$