sin, cos and tan are not 1-1. The \( x \)-axis is a horizontal line which crosses their graphs more than once. They are \textit{1-1} when restricted to the \textit{green} intervals.

For \( \sin \), the restricted interval is \([-\pi/2, \pi/2]\).

For \( \cos \), the restricted interval is \([0, \pi]\).

For \( \tan \), the restricted interval is \((-\pi/2, \pi/2)\).
Inverse trigonometric functions

\( \sin^{-1}(x) = \) the \( \theta \in [-\pi/2, \pi/2] \) such that \( \sin(\theta) = x \),
\( \cos^{-1}(x) = \) the \( \theta \in [0, \pi] \) such that \( \cos(\theta) = x \),
\( \tan^{-1}(x) = \) the \( \theta \in (-\pi/2, \pi/2) \) such that \( \tan(\theta) = x \).

\( \sin^{-1}(x) \) and \( \cos^{-1}(x) \) have domain \([-1, 1]\).
\( \tan^{-1}(x) \) has domain \((-\infty, \infty)\).

**Notation**

\( \text{arcsin}(x) = \sin^{-1}(x) \)
\( \text{arccos}(x) = \cos^{-1}(x) \)
\( \text{arctan}(x) = \tan^{-1}(x) \)

**Warning.** \( \sin^{-1}(x) \neq 1/\sin(x) \).

\( \sin^{-1}(x) = \text{arcsin } x = \) the inverse
\( (\sin(x))^{-1} = \frac{1}{\sin x} = \) the reciprocal
To find $\sin(\theta)$,
draw the angle $\theta$,
find its point on the unit circle,
$\sin(\theta)$ is the height of this point.

To find $\sin^{-1}(x)$,
mark the height $x$ on the $y$-axis,
find the point to its right on the unit circle,
$\sin^{-1}(x)$ is the angle for this point.
To find $\cos(\theta)$,
draw the angle $\theta$,
find its point on the unit circle,  
$\cos(\theta)$ is the $x$-coordinate of this point.

To find $\cos^{-1}(x)$,
mark $x$ on the $x$-axis,
find the point above it on the unit circle,
$\cos^{-1}(x)$ is the angle for this point.
<table>
<thead>
<tr>
<th>Function</th>
<th>Value</th>
<th>Function</th>
<th>Value</th>
<th>Function</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin^{-1}(0))</td>
<td>0</td>
<td>(\cos^{-1}(1))</td>
<td>0</td>
<td>(\tan^{-1}(0))</td>
<td>0</td>
</tr>
<tr>
<td>(\sin^{-1}(1/2))</td>
<td>(\pi/6)</td>
<td>(\cos^{-1}(\sqrt{3}/2))</td>
<td>(\pi/6)</td>
<td>(\tan^{-1}(1/\sqrt{3}))</td>
<td>(\pi/6)</td>
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<tr>
<td>(\sin^{-1}(1/\sqrt{2}))</td>
<td>(\pi/4)</td>
<td>(\cos^{-1}(1/\sqrt{2}))</td>
<td>(\pi/4)</td>
<td>(\tan^{-1}(1))</td>
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<tr>
<td>(\sin^{-1}(\sqrt{3}/2))</td>
<td>(\pi/3)</td>
<td>(\cos^{-1}(1/2))</td>
<td>(\pi/3)</td>
<td>(\tan^{-1}(\sqrt{3}))</td>
<td>(\pi/3)</td>
</tr>
<tr>
<td>(\sin^{-1}(1))</td>
<td>(\pi/2)</td>
<td>(\cos^{-1}(0))</td>
<td>(\pi/2)</td>
<td>(\tan^{-1}(0))</td>
<td>(0)</td>
</tr>
</tbody>
</table>

- \(\arcsin(-1/2)\)
- \(\arcsin(-1/\sqrt{2})\)
- \(\arccos(-1/2)\)
- \(\cos^{-1}(2)\)
- \(\arctan(-1)\)
- \(\arctan(-\sqrt{3})\)
- \(\cos(\tan^{-1}(-1)) = 1/\sqrt{2}\)
Inverses undo each other, hence
\[ \sin^{-1}(\sin \theta) = \theta \text{ if } \theta \in [-\pi/2, \pi/2] \]
\[ \sin(\sin^{-1} x) = x \text{ if } x \in [-1, 1] \]
\[ \cos^{-1}(\cos \theta) = \theta \text{ if } \theta \in [0, \pi] \]
\[ \cos(\cos^{-1} x) = x \text{ if } x \in [-1, 1] \]
\[ \tan^{-1}(\tan \theta) = \theta \text{ if } \theta \in (-\pi/2, \pi/2) \]
\[ \tan(\tan^{-1} x) = x \text{ for any } x \]

**RECALL.**  \( \cos(-x) = \cos x, \quad \sin(\pi - x) = \sin x. \)

If the angle is outside the restricted range, rewrite it in terms of an angle in the restricted range.

- \( \cos(\cos^{-1} \left( \frac{4}{5} \right)) \)
- \( \sin(\sin^{-1} \left( \frac{5}{4} \right)) \)
- \( \cos^{-1}(\cos \left( \frac{4\pi}{5} \right)) \)
- \( \sin^{-1}(\sin(-\frac{\pi}{5})) \)
\[ \sin^{-1}(\sin \theta) = \theta \text{ if } \theta \in [-\pi/2, \pi/2] \quad \sin(\sin^{-1} x) = x \text{ if } x \in [-1, 1] \]
\[ \cos^{-1}(\cos \theta) = \theta \text{ if } \theta \in [0, \pi] \quad \cos(\cos^{-1} x) = x \text{ if } x \in [-1, 1] \]
\[ \tan^{-1}(\tan \theta) = \theta \text{ if } \theta \in (-\pi/2, \pi/2) \quad \tan(\tan^{-1} x) = x \text{ for any } x \]

**RECALL.** \( \cos(-x) = \cos x, \quad \sin(\pi - x) = \sin x. \)

If the angle is outside the restricted range, rewrite it in terms of an angle in the restricted range.

- \( \cos^{-1}(\cos(-\frac{4\pi}{5})) \)
- \( \cos^{-1}(\cos(-\frac{\pi}{5})) \)
\[
\begin{align*}
\sin^{-1}(\sin \theta) &= \theta \text{ if } \theta \in [-\pi/2, \pi/2] \\
\sin(\sin^{-1} x) &= x \text{ if } x \in [-1, 1] \\
\cos^{-1}(\cos \theta) &= \theta \text{ if } \theta \in [0, \pi] \\
\cos(\cos^{-1} x) &= x \text{ if } x \in [-1, 1] \\
\tan^{-1}(\tan \theta) &= \theta \text{ if } \theta \in (-\pi/2, \pi/2) \\
\tan(\tan^{-1} x) &= x \text{ for any } x
\end{align*}
\]

**RECALL.** \(\cos(-x) = \cos x, \quad \sin(\pi - x) = \sin x.\)

If the angle is outside the restricted range, rewrite it in terms of an angle in the restricted range.

- \(\sin^{-1}(\sin \frac{4\pi}{5})\)
- \(\sin^{-1}(\sin \frac{5\pi}{7})\)

\[\text{.} \quad \text{.} \]
\[ \sin^{-1}(\sin(-\frac{4\pi}{5})) \]
**Theorem.** For \( x \in [-1, 1] \), \( \cos(\sin^{-1}x) = \sqrt{1 - x^2} \)
\[
\sin(\cos^{-1}x) = \sqrt{1 - x^2}
\]

**Proof of 1st.** Recall: \( \cos^2 \theta = 1 - \sin^2 \theta \).
\[
\therefore \quad \cos \theta = \pm \sqrt{1 - \sin^2 \theta}
\]
\[
\cos(\sin^{-1}x) = \pm \sqrt{1 - (\sin(\sin^{-1}x))^2}
\]
\[
= \sqrt{1 - x^2}
\]

“+” since \( \sin^{-1}(x) \in [-\pi/2, \pi/2] \implies \cos(\sin^{-1}(x)) \geq 0. \)

\[ \sin(\sin^{-1}(\frac{1}{5})) \]
Theorem. For $x \in [-1, 1]$, 
\[
\cos(\sin^{-1} x) = \sqrt{1 - x^2} \\
\sin(\cos^{-1} x) = \sqrt{1 - x^2}
\]
- $\sin(\cos^{-1}(\frac{1}{3}))$  
  $\cos(\sin^{-1}(\frac{1}{3}))$  
  ... $2\sqrt{6}/5$
\[
\tan(\sin^{-1}(\frac{1}{3})) \quad \ldots \quad \frac{1}{2\sqrt{2}}
\]

\[
\sec[\sin^{-1}(-1) - \cos^{-1}(1)] \quad \ldots \quad \text{undefined}
\]
Find angle $A$.

Find angle $B$. 
**Fact:** $\sin^{-1} x + \cos^{-1} x = \pi/2$

These are complementary angles. If you know one, you can solve for the other.

Given: $\cos^{-1} x = \frac{\pi}{3}$.

Find $\sin^{-1} x$. More than one answer may be correct.
To test if your calculator is in radian or degree mode, calculate \( \sin(1) \).

\[
\begin{align*}
\sin(1^\circ) & \approx 0.017 \quad \text{degree mode} \\
\sin(1 \text{ rad}) & \approx 0.84 \quad \text{radian mode}
\end{align*}
\]

Bookstore has cheap $20 trig. calculators.

[www.math.hawaii.edu/140](http://www.math.hawaii.edu/140) has downloadable calculators.

In Google or WolframAlpha

<table>
<thead>
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<tbody>
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<td>( \sin(1) )</td>
<td>0.84</td>
</tr>
<tr>
<td>( \sin(1 \text{ rad}) )</td>
<td>0.84</td>
</tr>
<tr>
<td>( \sin(1 \text{ degree}) )</td>
<td>0.02</td>
</tr>
<tr>
<td>( \arcsin(1) )</td>
<td>1.57</td>
</tr>
<tr>
<td>( \arcsin(1) \text{ in radians} )</td>
<td>1.57 in radians</td>
</tr>
<tr>
<td>( \arcsin(1) \text{ in degrees} )</td>
<td>90 degrees</td>
</tr>
</tbody>
</table>