A circle is the set of all points such that the distance to a center point is some constant $r$.

**Definition.** An *ellipse* is the set of all points such that the sum of the distances to two *focus* points = the largest diameter = the distance between the vertices.

The *vertices* are the two points farthest apart.

The *major axis* runs between the two *vertices*. It has the largest diameter.

The *minor axis* is a perpendicular bisector of the major axis. It has the smallest diameter.

Mark the foci with a compass.
- Set your compass to a major radius.

- Put the point end on a minor axis endpoint.

- Draw an arc intersecting the major radius at the two foci.

By the Pythagorean Theorem,

\[ a^2 = b^2 + c^2 \]
\[ a^2 - b^2 = c^2 \]
\[ \therefore c^2 = a^2 - b^2 \]
\[ \therefore c = \sqrt{a^2 - b^2} \]
\( a = \text{major radius} = \frac{1}{2} \) the major axis length.
\( b = \text{minor radius} = \frac{1}{2} \) the minor axis length.
\( c = \text{focal radius} = \frac{1}{2} \) the distance between the foci.

**Theorem.** \( a^2 = b^2 + c^2 \).
\[
\therefore \quad c = \sqrt{a^2 - b^2}.
\]

The equation of a circle with center \((0, 0)\) and radius \(r\) is \(x^2 + y^2 = r^2\). Dividing both sides by \(r^2\) gives
\[
\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1.
\]

The equation of an ellipse with center \((0, 0)\) and horizontal radius \(a\) and vertical radius \(b\) is
\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.
\]
For ellipse equations, put the larger radius first.

**Theorem.** For \(a \geq b > 0\) (larger denominator first), the graphs of

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{and} \quad \frac{y^2}{a^2} + \frac{x^2}{b^2} = 1
\]

are ellipses.

(0, 0) is the center.

\(a\) = the major radius.

\(b\) = the minor radius.

\(c = \sqrt{a^2 - b^2}\) = the focal radius.

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{is a horizontal ellipse.}
\]

\(x\) has the bigger radius.

foci: \((c, 0)\) and \((-c, 0)\)

major axis: \((-a, 0)(a, 0)\)

minor axis: \((0, -b)(0, b)\)
\( \frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \) is a **vertical** ellipse. 

\( y \) has the **bigger** radius \( a \).

- **Axes:**
  - **Minor axis:** \((-b,0)(b,0)\)
  - **Major axis:** \((0,-a)(0,a)\)
  - **Foci:** \((0,-c)\) and \((0,c)\)

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To graph:

- Complete the squares if necessary.
- Get 1 on the right.

- Write the equation in one of the two **ellipse forms**.
  The term with the larger denominator \( a \) comes first.

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{horizontal ellipse, } a > b
\]

\( x \) has the larger denominator.

\[
\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \quad \text{vertical ellipse, } a > b
\]

\( y \) has the larger denominator.

- Add any needed horizontal or vertical shifts.
Find the major and minor axes and foci; graph:
- \( x^2 + 9y^2 = 9 \)
- \( 4x^2 + y^2 = 4 \)

Write the equation in ellipse form.

Find \( a \).
Find \( b \).
Find \( c \).

Find the correct shape.

(A) horizontal ellipse  (B) vertical ellipse
(C) horizontal parabola  (D) vertical parabola

(E) #
Continued.

\[ x^2 + 9y^2 = 9. \]

Find the major axis.
\((-a, 0)(a, 0) \text{ or } (0, -a)(0, a)\)

Find the minor axis.
\((-b, 0)(b, 0) \text{ or } (0, -b)(0, b)\)

Find foci. \((\pm c, 0) \text{ or } (0, \pm c)\)

Graph.

\[ 4x^2 + y^2 = 4. \]
THEOREM  In any equation, replacing each
  \[ x \text{ by } x-a \rightarrow \text{right } a \text{ units} \]
  \[ x \text{ by } x+a \rightarrow \text{left } a \text{ units} \]
  \[ y \text{ by } y-b \uparrow \text{up } b \text{ units} \]
  \[ y \text{ by } y+b \downarrow \text{down } b \text{ units} \]

Shifting a graph also shifts its vertices, foci, and axes.
Find the major and minor axes and the foci:
\[ 4x^2 + y^2 + 2y = 3. \quad \text{and} \quad 16x^2 - 96x + 25y^2 = 256. \]

Write equation in ellipse form.

Find \( a \).
Find \( b \).
Find \( c \).
Find the shift.

Find the correct shape.
Continued.

- $4x^2 + y^2 + 2y = 3$.

Find the major axis.

- $16x^2 - 96x + 25y^2 = 256$.

Find the minor axis.

Find foci.
Continued.

- $4x^2 + y^2 + 2y = 3$

Find the graph.
Identify the type of graph:
\[ y^2 + 2y = 4x - 5. \]
\[ y^2 + 2y = 3 - 4x^2. \]
\[ y^2 + 2y = 3 + 4x^2. \]

On the final, you get almost as many points for writing the foci and axes in symbolic form, \((a, b)(c, d)\), as for the graph.