Math 241 Discussion 2

**FACTS** $\infty \pm a = \infty$, $a \infty = \infty$ or $-\infty$, $\sqrt{-\infty} = \infty^2 = \infty^3 = \infty$, $a/\infty = 0$.

The following terms are ordered according to their degree (their power or exponent). Coefficients are ignored.

$10x^{-4}$, $1/x = x^{-1}$, $1 = x^0$, $\sqrt{x} = x^{1/2}$, $x = x^1$, $6x^2$, $-3x^4$.

Hence $2\sqrt{x}$ has higher power than 7, 2 has higher power than $\frac{4}{x}$, $-x$ has higher power than $\sqrt{3x}$.

**INFINITE LIMIT CALCULATION** For infinite limits involving powers of $x$ (e.g., $\frac{\sqrt{x^2-5}}{\sqrt{x+3}}$), factor out the highest power of $x$ in any factor consisting of a sum of powers of $x$, simplify, then plug in the value $x$ approaches.

- $\lim_{x \to \infty} \frac{(2x^2 + 3x - 1)(x+4)}{(x^2 - 3)} = \lim_{x \to \infty} \frac{[x^2(2+\frac{3}{x}-\frac{1}{x^2})](x+4)}{x^2(1-\frac{3}{x^2})}$
  $= \lim_{x \to \infty} \frac{[x^2][x]}{x^2} \cdot \lim_{x \to \infty} \frac{(2+\frac{3}{x}-\frac{1}{x^2})(1+\frac{4}{x})}{(1-\frac{3}{x^2})}$
  $= \lim_{x \to \infty} \frac{x}{1} \cdot \lim_{x \to \infty} \frac{(-\infty)(2)(1)}{(1-\frac{3}{x^2})} = \infty \cdot \infty = -\infty$

- $\lim_{x \to \infty} \frac{1 - \frac{3}{x}}{(2+\sqrt{x})} = \lim_{x \to \infty} \frac{\frac{1}{x}(1-\frac{3}{x})}{\sqrt{x + \frac{2}{x} + 1}} = \lim_{x \to \infty} \frac{\frac{1}{x}}{(\sqrt{x} + \frac{2}{x})} = \infty \cdot \frac{1}{1} = 0$

Note: In any sum of powers, terms not of highest power go to 0. Might as well eliminate them at the beginning.

**SHORTCUT INFINITE LIMIT CALCULATION.** For infinite limits involving powers of $x$,

Within each factor consisting of a sum of powers of $x$:

- omit terms not of highest power (what is left is called the leading term),
- simplify,
- stick in $\infty$ or $-\infty$.

- $\lim_{x \to \infty} \frac{(2x^2 + 3x - 1)(x+4)}{(x^2 - 3)} = \lim_{x \to \infty} \frac{2x^2(x)}{(x^2)} = \lim_{x \to \infty} 2(x) = 2(-\infty) = -\infty$

- $\lim_{x \to \infty} \frac{1 - \frac{3}{x}}{(2+\sqrt{x})} = \lim_{x \to \infty} \frac{1}{(\sqrt{x})} = \lim_{x \to \infty} \frac{1}{x^{\frac{1}{2}}} = \infty^{\frac{1}{2}} = 0$

Groupwork problems. To get credit, stay with your group until the problems have been completed or until the end of the period. Everyone still with the group gets the lowest score. No credit if you are not with a group.

Calculate the following. Show your work: (1) show the rewritten formula with only the leading terms, (2) show the result of plugging in $\pm \infty$, (3) show the final answer.

48(1). $\lim_{x \to \infty} \frac{2x^3 + 7}{x^3 - x^2 + x + 7}$. Ans: 2

50(1). $\lim_{x \to \infty} \frac{3x + 7}{x^2 - 2}$. Ans: 0

58(1). $\lim_{x \to \infty} \frac{2 + \sqrt{x}}{2 - \sqrt{x}}$. Ans: -1

60(1). $\lim_{x \to \infty} \frac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}}$. Ans: $\infty$