Graphs continued 35 min lecture

Know these graphs.

- $y = \frac{1}{x}$
- $y = -\frac{1}{x}$
- $y = \frac{1}{x^2}$
- $y = -\frac{1}{x^2}$
- $y = \frac{1}{x^3}$
- $y = -\frac{1}{x^3}$

○ For odd degree roots or vertical asymptotes, one side is positive and the other is negative. Like $-x$, $x^3$, $\frac{1}{x}$, $-\frac{1}{x}$.

○ For even degree roots or vertical asymptotes, both sides are positive or both are negative, as $x^2$, $\frac{1}{x^2}$, $-\frac{1}{x^2}$.

Getting the leading term: in each factor which is a sum of powers of $x$, delete all but the highest power, simplify.

Find the leading terms:

- $\frac{x\sqrt{x}}{1 - x}$ → $\frac{x\sqrt{x}}{-x} = -\sqrt{x}$
- $\sqrt{x^4 - 1}$
  - (A) $x^2 - 1$
  - (B) $x^2$
  - (C) $x^4 - 1$
  - (D) $x^4$
  - (E) # (none of these)

- $\frac{x(1 - 3x)}{(1 - x)^2} \rightarrow \frac{x(-3x)}{(-x^2)} = -\frac{3x^2}{x^2} = -\frac{3}{x^2}$
- $\frac{(1 - 2x)^3}{(1 - x)}$
  - (A) 2
  - (B) $2x$
  - (C) $x^2$
  - (D) $8x^2$
  - (E) $-8x^3$

Recall:

**Theorem** As $x \to \pm \infty$, the graph resembles the *leading term*. $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ equal the limit of the leading term.

**Common error** If your graph crosses the $x$-axis anywhere other than the $x$-intercepts you have an error.
Roughly, a function is **continuous** if it makes no jumps, i.e., its graph has no breaks over points where it is defined.

- **Continuous functions**

  ![Continuous functions]

- **Discontinuous functions**

  ![Discontinuous functions]

  * f is **continuous at a point** c, if there is no break in the graph at c iff \( \lim_{x \to c} f(x) = f(c) \)

  * If there is a break at c
    - c is a **removable discontinuity** if the break can be removed by moving the value \( f(c) \) up or down. Iff \( \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) \neq f(c) \)
    - c is an **essential discontinuity** otherwise. Iff \( \lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x) \)
    - f is **continuous from the left (right)** at c if you don't have to jump to get to the value \( f(c) \) as you approach c from the left (from the right).

- **Classify the discontinuities.**

  ![Classify the discontinuities.]

- f is **continuous on an interval** \([a,b]\) iff there are no jumps inside \([a,b]\)
  
  - f is
    - continuous from the right at a
    - continuous from the left at b
    - continuous at all other points in \([a,b]\).

- **On which intervals is f continuous?**
  - (A) Continuous
  - (B) Not continuous
    - \([p,q]\)
    - \([r,s]\)
    - \([s,t]\)

**Fact** If f is built up from

\(+, -, \cdot, \div, \sqrt[n]{x}, \log(x), a^x, \sin, \cos, \tan\) and not defined piecewise — then f is continuous wherever it is defined.

Use this fact to solve the last problem (question 4) of MML Hw 8.