Velocity and Acceleration

“Rate of change” without any “with respect to”, is understood to mean “with respect to time”.

**Definitions** Suppose a point moves up and down along the y-axis (or more generally, forward/backward along any straight line). Let \( s(t) \), \( v(t) \), \( a(t) \) be the position, velocity, acceleration at time \( t \).

\[ v(t) = \text{the absolute value of velocity} = \text{its speed} \text{ at time } T. \]

\[ s(b) - s(a) = \text{the difference between its final and initial positions} = \text{its displacement} \text{ over a period } [a, b] \text{ of time}. \]

\[ \frac{s(b) - s(a)}{b - a} = \text{average rate of change} = \text{the average velocity}. \]

**Theorem**

Velocity = rate of change of position.

\[ v(t) = \frac{ds}{dt} = s'(t) \]

Acceleration = rate of change of velocity.

\[ a(t) = \frac{dv}{dt} = v'(t) = (s'(t))' = \frac{d^2s}{dt^2} \]

An object is moving up/down/motionless (or forward/backward/stopped) iff its position is increasing/decreasing/constant iff the rate of change (slope) is positive/negative/zero iff the velocity \( v(t) \) is positive/negative/zero iff \( s'(t) > 0, < 0, = 0 \).

The velocity is increasing/decreasing/constant iff the rate of change of velocity is positive/negative/zero iff the acceleration \( a(t) \) is positive/negative/zero iff \( v'(t) > 0, < 0, = 0 \).

\[ s = t^3 - 3t \text{ for } t \in [-2, 2] \text{ is the position in meters on the y-axis at time } t \text{ seconds.} \]

Since \( s(t) = t^3 - 3t \text{ for } t \in [-2, 2] \)

\[ v(t) = s'(t) = (t^3 - 3t)' = 3t^2 - 3 \]

\[ a(t) = v'(t) = (3t^2 - 3)' = 6t \]

The displacement (distance between the start and end positions) = \( s(2) - s(-2) = (8 - 6) - (-8 - (-6)) = 2 - (-2) = 4 \) meters.

The average velocity \[ \frac{s(2) - s(-2)}{2 - (-2)} = \frac{4}{4} = 1 \text{ meters/second}. \]

Its velocity at the beginning = \( v(-2) = 3(-2)^2 - 3 = 9 \frac{m}{s} \)

Acceleration at the beginning = \( a(-2) = 6(-2) = -12 \frac{m}{s^2} \)

When does the object change direction? When it changes direction, its velocity is zero or undefined. Here \( v(t) = 0 \iff 3t^2 - 3 = 0 \iff t^2 - 1 = 0 \iff t = -1, 1 \) seconds.

When does it to reach its maximum height? It will be at its max or min height either at the initial time or the final time or when its velocity is 0 (or undefined).

The times of maximum height are \( t = -1, 2 \) seconds.

The max height is \( s(-1) = s(2) = 2 \) meters.

The times of minimum height are \( t = -2, 1 \) seconds.

The min height is \( s(-2) = s(1) = -2 \) (use format for max/mins)

Again, a point moves along the y-axis. The graph gives its height on the y-axis at a time t on the x-axis.

Find the **first** graph with a **positive velocity**.

Find the **second** graph with a **positive velocity**.

Find the **first** graph with a **negative velocity**.

Find the **first** graph with a **positive acceleration**.

Find the **first** graph with a **negative acceleration**.
An object’s position $s$ at time $t$ is plotted on the $y$-axis.

On what interval $[a, b]$ of time does the object move downward? $a = ?$ $b = ?$

On what interval $[a, b]$ of time is the object motionless? $a = ?$ $b = ?$

On what interval $[a, b]$ of time does the object have its greatest speed? $a = ?$ $b = ?$ Steepest positive or negative.

On what interval $[a, b]$ of time does the object have its greatest velocity? $a = ?$ $b = ?$ Note, positive > negative.

An object’s velocity $v$ at time $t$ is plotted on the $y$-axis.

On what interval $[a, b]$ of time does the object move downward? $a = ?$ $b = ?$

On what interval $[a, b]$ is the velocity decreasing? $a = ?$ $b = ?$

When the object moving at its greatest velocity? When is the object motionless?