$x^y = \text{base}^{\text{exponent}}$. For the power rule $(x^n)' = nx^{n-1}$ the base must be a single variable and the exponent a constant.

$e^x' = e^x \neq xe^{x-1}$ — for the power rule, the exponent is a constant

$((2x + 3)^7)' \neq 7(2x + 3)^6$ — for the power rule, the base is a single variable. To apply the power rule here, one must use the chain rule (covered later).

These two occur often, memorize them.

\[
(\frac{1}{x})' = \frac{-1}{x^2} \quad \quad (\sqrt{x})' = \frac{1}{2\sqrt{x}}
\]
Velocity and Acceleration

“Rate of change” without any “with respect to”, is understood to mean “with respect to time”.

**Definitions** Suppose a point moves up and down along the $y$-axis (or more generally, forward/backward along any straight line). Let $s(t), v(t), a(t)$ be the *position*, *velocity*, *acceleration* at time $t$.

$|v(t)| =$ the absolute value of velocity = its *speed* at time $T$.

$s(b) - s(a)$

= the difference between its final and initial positions

= its *displacement* over a period $[a, b]$ of time.

$s(b) - s(a) \over b - a$ = average rate of change = the *average velocity*.

**Theorem**

Velocity = rate of change of position (with respect to time).

$v(t) = {ds \over dt} = s'(t)$

Acceleration = rate of change of velocity.

$a(t) = {dv \over dt} = v'(t) = (s'(t))' = {d^2 s \over dt^2}$
An object is moving up/down/motionless (or forward/backward/stopped) iff its position is increasing/decreasing/constant iff the rate of change (slope) is positive/negative/zero iff the velocity $v(t)$ is positive/negative/zero iff $s'(t) > 0, < 0, = 0$.

The velocity is increasing/decreasing/constant iff the rate of change of velocity is positive/negative/zero iff the acceleration $a(t)$ is positive/negative/zero iff $v'(t) > 0, < 0, = 0$. 
\[ s = t^3 - 3t \text{ for } t \in [-2, 2] \] is the position in meters on the y-axis at time \( t \) seconds. -2 is the start time, 2 the stop time.

Since \( s(t) = t^3 - 3t \) for \( t \in [-2, 2] \)

\[
\begin{align*}
v(t) &= s'(t) = (t^3 - 3t)' = 3t^2 - 3 \\
a(t) &= v'(t) = (3t^2 - 3)' = 6t
\end{align*}
\]

The displacement (distance between the start and end positions) =

\[
s(2) - s(-2) = (8 - 6) - (-8 - (-6)) = 2 - (-2) = 4 \text{ meters.}
\]

The average velocity \[
\frac{s(2) - s(-2)}{2 - (-2)} = \frac{4}{4} = 1 \text{ meters/second.}
\]
Its velocity at the beginning $= v(-2) = 3(-2)^2 - 3 = 9 \frac{m}{s}$

Acceleration at the beginning $= a(-2) = 6(-2) = -12 \frac{m}{s^2}$

When it changes direction, its velocity is zero or undefined. Here $v(t) = 0 \iff 3t^2 - 3 = 0 \iff t^2 - 1 = 0 \iff t = -1, 1$ seconds.
When does it reach its maximum height?

It will be at its max or min height either at the initial time or the final time or when its velocity is 0 (or undefined).

The times of maximum height are \( t = -1, 2 \) seconds.

The max height is \( s(-1) = s(2) = 2 \) meters.

The times of minimum height are \( t = -2, 1 \) seconds.

The min height is \( s(-2) = s(1) = -2 \) (use format for max/mins).
Again, a point moves along the $y$-axis. The graph gives its height on the $y$-axis at a time $t$ on the $x$-axis.

Find the **first** graph with a **positive velocity**.

Find the **second** graph with a **positive velocity**.

Find the **first** graph with a **negative velocity**.

Find the **first** graph with a **positive acceleration**.

Find the **first** graph with a **negative acceleration**.
An object’s position $s$ at time $t$ is plotted on the $y$-axis.

On what interval $[a, b]$ of time does the object move downward? $a = ?$  $b = ?$

On what interval $[a, b]$ of time is the object motionless? $a = ?$  $b = ?$

On what interval $[a, b]$ of time does the object have its greatest speed? $a = ?$  $b = ?$  Steepest positive or negative.

On what interval $[a, b]$ of time does the object have its greatest velocity? $a = ?$  $b = ?$  Note, positive $>$ negative.
An object’s velocity $v$ at time $t$ is plotted on the $y$-axis.

On what interval $[a, b]$ of time does the object **move downward**?

$\quad a = ? \quad b = ?$

On what interval $[a, b]$ is the velocity **decreasing**?

$\quad a = ? \quad b = ?$

When the object moving at its **greatest velocity**?

When is the object **motionless**? □