**Math 241     Lecture 17**

**FACT** If a line has slope \(m\), a perpendicular line will have slope \(-\frac{1}{m}\).

![Tangent and normal lines](image)

**Definition** The normal to a curve at a given point is the line through that point which is perpendicular to the tangent.

If a tangent has slope \(m\), the normal has slope \(-\frac{1}{m}\).

\[m = \frac{dy}{dx} \text{ rotates to } -\frac{dx}{dy} = -\frac{1}{dy/dx} = -\frac{1}{m}.\]

The tangent to a curve at \((1, 2)\) is \(y = 2x\) . Find the equation for the normal at \((1, 2)\).

The tangent slope is \(2\). The normal answer is not \(m = 2\). Use point-slope equation: 

\[y - y_0 = -\frac{1}{m} (x - x_0).\]

(A) \(y = -2x + \frac{3}{2}\)

(B) \(y = -2x + 4\)

(C) \(y = -\frac{1}{2}x + \frac{5}{2}\)

(D) \(y = -\frac{1}{2}x + 4\)

(E) #

**Implicit Differentiation**

Suppose \(y = f(x)\) . Then

\[dy/dx = y' = f'(x)\]

\[\left[f(x)^4\right]' = 4(f(x))^3 f'(x)\]

\[\frac{d}{dx} y^4 = 4y^3 \frac{dy}{dx}\]

\[\frac{d}{dy} y^4 = 4y^3\]

Find \(\frac{d}{dx} y^3\)

(A) \(3y^2\)

(B) \(3\left(\frac{dy}{dx}\right)^2\)

(C) \(3y^2 \frac{dy}{dx}\)

(D) \(3\left(\frac{dy}{dx}\right)^2\)

(E) \(3\left(\frac{dy}{dx}\right)^2 y\)

- Find equations for the tangent and normal for \(x^2 + xy + 2y^2 = 28\) at \((-2, -3)\).
  - In WolframAlpha, type: \(x^2+xy+2y^2=28\) normal at \((-2,-3)\)

- Differentiate w.r.t. \(x\).
- Solve for \(dy/dx\).
- Replace \(x\) and \(y\) with -2 and -3.

\[x^2 + xy + 2y^2 = 28\]

\[2x + (1 \cdot y + x \frac{dy}{dx}) + 4y \frac{dy}{dx} = 0\]

\[(x + 4y) \frac{dy}{dx} = -(2x + y)\]

\[\frac{dy}{dx} = \frac{-(2x + y)}{x + 4y}\]

\[\frac{dy}{dx} \text{ at } (-2, 3) = \frac{-(2(-2) + (3))}{(-2) + 4(-3)} = \frac{-3}{-14} = -\frac{1}{2}\]

The tangent equation is \(y - y_0 = m(x - x_0)\) where \((x_0, y_0) = (-2, -3)\) and \(m = -1/2\).

Tangent equation: 

\[y - (-3) = -\frac{1}{2}(x - (-2))\]

\[y + 3 = -\frac{1}{2}x + 1\]

\[y = -\frac{1}{2}x - 4\]

Normal slope: \(-\frac{1}{m} = -\frac{1}{-1/2} = 2\)

Normal equation: 

\[y - (-3) = 2(x - (-2))\]

\[y + 3 = 2x + 4\]

\[y = 2x + 1\]