Rate of change problems

- A 6’ ladder leans against a wall. The base is pulled away at 1/2 ft/sec. How fast is the top falling when the base is 5’ from the wall?

**Picture & variables**

<table>
<thead>
<tr>
<th>Want</th>
<th>(A) dy/dt</th>
<th>(B) -dy/dt</th>
<th>(C) dy/dt</th>
<th>(D) -dy/dt</th>
<th>(E) #</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given x² + y² = 6² and the second given is</td>
<td>(A) y = 5</td>
<td>(B) x = 5</td>
<td>(C) dx/dt = 1/2</td>
<td>(D) dy/dt = -1/2</td>
<td>(E) #</td>
</tr>
</tbody>
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**Diff.** Differentiate the equation.

(A) 2x + 2y = 0     (B) 2x + 2y dy/dt = 0

(C) 2x dx/dt + 2y = 0     (D) 2x dx/dt + 2y dy/dt = 0     (E) #

Replace dx/dt with its value and then solve for dy/dt.

(A) \[ \frac{x}{y} \]     (B) \[ -\frac{x}{2y} \]     (C) \[ \frac{y}{2x} \]     (D) \[ -\frac{y}{x} \]     (E) #

**Ans.** dy/dt | x=5, y=?

y² + 5² = 6², y = \[ \sqrt{36-25} = \sqrt{11} \]

**Final Ans.** Rate of fall = \[ -\frac{dy}{dt} \mid _{x=5, y=\sqrt{11}} = \text{? ft/sec} \]

(A) \[ \frac{1}{5\sqrt{11}} \]     (B) \[ -\frac{1}{5\sqrt{11}} \]     (C) \[ -\frac{5}{\sqrt{11}} \]     (D) \[ \frac{5}{2\sqrt{11}} \]     (E) #

- A conical cup has radius 2" and height 6". It leaks water at a rate of 5 cubic inches/min. How fast is the water level falling when the level is 3"?

The volume V of a cone of radius r, height h is \[ V = \frac{\pi}{3} r^2 h. \]

**Picture & variables**

<table>
<thead>
<tr>
<th>Want</th>
<th>(A) [ \frac{dh}{dt} \mid _{h=3} ]</th>
<th>(B) [ -\frac{dh}{dt} \mid _{h=3} ]</th>
<th>(C) [ \frac{dr}{dt} \mid _{h=3} ]</th>
<th>(D) [ -\frac{dr}{dt} \mid _{r=3} ]</th>
<th>(E) #</th>
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</thead>
<tbody>
<tr>
<td>Given [ V = \frac{\pi}{3} r^2 h, ] [ \frac{dV}{dt} = -5 ], find the third given which relates r and h.</td>
<td>(A) h = 6</td>
<td>(B) h = 3</td>
<td>(C) r = 2</td>
<td>(D) [ \frac{r}{h} ] = [ \frac{2}{6} ]</td>
<td>(E) #</td>
</tr>
</tbody>
</table>

**Eq. Want eq. in V (known rate of change) and h (wanted rate of change) and nothing else -- no r. Need r in h.**

\[ V = \frac{\pi}{3} r^2 h \]

To get an equation in just V and h we need to write r in terms of h.

\[ \frac{r}{h} = \frac{2}{6}, \] \[ 6r = 2h, \] \[ 3r = h, \] \[ r = \frac{h}{3} \]

**Write V in terms of h.**

\[ V = \frac{\pi}{27} h^3 \]

**Diff.** Differentiate both sides of the equation \[ V = \ldots \]

\[ \frac{dV}{dt} = \]

(A) \[ 3\pi h^2 \]     (B) \[ 3\pi h^2 \frac{dh}{dt} \]     (C) \[ \frac{\pi h^2}{9} \]     (D) \[ \frac{\pi h^2}{9} \frac{dh}{dt} \]     (E) #

**Ans.** Find \[ \frac{dh}{dt} \mid _{h=3}. \]

In the equation \[ \frac{dV}{dt} = \ldots \] set \[ \frac{dV}{dt} = -5. \]

Solve for \[ \frac{dh}{dt} \] and set \[ h = 3. \]

**Final Ans:** Water is falling at \[ -\frac{dh}{dt} \mid _{h=3} = \text{? inches/min} \]

(A) \[ 5\pi \]     (B) \[ \frac{1}{5\pi} \]     (C) \[ 3\pi \]     (D) \[ \frac{1}{3\pi} \]     (E) #

- A 6’ man walks toward from a 10’ streetlight at 6 feet/second. Find the rate of change of the shadow’s length when he is 6’ from the base of the light. ... -9 ft/sec