Rate of change problems

- A 6' ladder leans against a wall. The base is pulled away at 1/2 ft/sec. How fast is the top falling when the base is 5' from the wall?

*Picture & variables*

\[ x \quad \rightarrow \quad 1/2 \]

\[ y \quad 6 \]

Want

(A) \( \frac{dy}{dt} \)  \hspace{1cm} (B) \( \frac{-dy}{dt} \)  \hspace{1cm} (C) \( \frac{dy}{dt} \big|_{x=5} \)  \hspace{1cm} (D) \( \frac{-dy}{dt} \big|_{x=5} \)  \hspace{1cm} (E) \#

Given \( x^2 + y^2 = 6^2 \) and the second given is

(A) \( y = 5 \)  \hspace{1cm} (B) \( x = 5 \)  \hspace{1cm} (C) \( \frac{dx}{dt} = \frac{1}{2} \)  \hspace{1cm} (D) \( \frac{dy}{dt} = -\frac{1}{2} \)  \hspace{1cm} (E) \#
Eq. \( x^2 + y^2 = 36 \)

Diff. Differentiate the equation.

(A) \( 2x + 2y = 0 \)    (B) \( 2x + 2y \frac{dy}{dt} = 0 \)

(C) \( 2x \frac{dx}{dt} + 2y = 0 \)    (D) \( 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \)    (E) #

Replace \( \frac{dx}{dt} \) with its value and then solve for \( \frac{dy}{dt} \)

\[
\frac{dy}{dt} = \]

(A) \( \frac{x}{y} \)    (B) \( \frac{-x}{2y} \)    (C) \( \frac{y}{2x} \)    (D) \( \frac{-y}{x} \)    (E) #

\[
\frac{dy}{dt} \bigg|_{x=5, \ y=\ ?}
\]

\( y^2 + 5^2 = 6^2 \), \( y = \sqrt{36 - 5^2} = \sqrt{11} \)

Ans. Rate of fall = \( \frac{-dy}{dt} \bigg|_{x=5, \ y=\sqrt{11}} = ? \) ft/sec

(A) \( \frac{1}{5\sqrt{11}} \)    (B) \( \frac{-1}{5\sqrt{11}} \)    (C) \( \frac{-5}{\sqrt{11}} \)    (D) \( \frac{5}{2\sqrt{11}} \)    (E) #
A conical cup has radius 2" and height 6". It leaks water at a rate of 5 cubic inches/min. How fast is the water level falling when the level is 3"? The volume $V$ of a cone of radius $r$, height $h$ is $V = \frac{\pi}{3} r^2 h$

**Picture & variables**

![Diagram of a conical cup with dimensions 2" radius, 6" height, and 5 cubic inches/min leak rate]

Want

(A) $\frac{dh}{dt} \big|_{h=3}$  (B) $-\frac{dh}{dt} \big|_{h=3}$  (C) $\frac{dr}{dt} \big|_{h=3}$  (D) $-\frac{dr}{dt} \big|_{r=3}$  (E) #

Given $V = \frac{\pi}{3} r^2 h$, $\frac{dV}{dt} = -5$, find the third given which is an equation between $r, h$.

(A) $h = 6$  (B) $h = 3$  (C) $r = 2$  (D) $\frac{r}{h} = \frac{2}{6}$  (E) #
Eq. Want eq. in $V$ (known rate of change) and $h$ (wanted rate of change) and nothing else -- no $r$. Need $r$ in $h$.

$V = \frac{\pi}{3} r^2 h$ involves $r$ and $h$.

To get an equation in just $V$ and $h$ we need to write $r$ in terms of $h$.

$6r = 2h, \ 3r = h, \ r = \frac{h}{3}$

Write $V$ in terms of $h$. $V =$

(A) $\frac{\pi}{27} h^3$  (B) $\frac{\pi}{27} h^2$  (C) $3\pi h^2$  (D) $-3\pi h^2$  (E) #

Diff. Differentiate both sides of the equation $V =$ ...

$\frac{dV}{dt} =$

(A) $3\pi h^2$  (B) $3\pi h^2 \frac{dh}{dt}$  (C) $\frac{\pi h^2}{9}$  (D) $\frac{\pi h^2}{9} \frac{dh}{dt}$  (E) #
Ans. Find $\frac{dh}{dt}|_{h=3}$.

In the equation $\frac{dV}{dt} = ...$ set $\frac{dV}{dt} = -5$.

Solve for $\frac{dh}{dt}$ and set $h = 3$.

**Answer:** Water is falling at $-\frac{dh}{dt}|_{h=3} = ?$ inches/min

(A) $5\pi$  (B) $\frac{1}{5\pi}$  (C) $3\pi$  (D) $\frac{1}{3\pi}$  (E) #

- A 6’ man walks toward from a 10’ streetlight at 6 feet/second. Find the rate of change of the shadow’s length when he is 6’ from the base of the light. ... -9 ft/sec