Recall: $df$ estimates the error in calculating $f(x)$ when the error in $x$ is $dx$.

- When measuring $x$ near 1, how much error $dx$ is allowed in your measurement of $x$ if you wish the error $\Delta f$ in $f(x) = x^{13}$ to be at most 10%? I.e., 10% of $f(1)$.

First find the differential form.

$$
\frac{df}{dx}|_{x=1} = (13x^{12})' |_{x=1} = (13x^{12}) |_{x=1} = 13
$$

Solve for $df$ to get the differential form:

$$
df = 13dx
$$

The error must be at most 10% of $f(1)$

$$
\Delta f \leq (0.1)f(1) \quad \Rightarrow \quad \Delta x \leq (0.1) \frac{1}{f(1)} = (0.1)(1)^{13} = 0.1
$$

Thus, $dx < 0.1/13 = 0.0077$

$x$ must be measured with error $\leq 0.0077$

- A surveyor estimates the height $h$ of a cliff by measuring its angle $\theta$ of inclination from a point 2 miles from its base. He measures $\theta$ to be $65^\circ$. His height estimate must be accurate to within .01 of a mile. Find the maximum allowed error $dl$ in his angle measurement.

$$
\tan \theta = \frac{h}{2} \quad \Rightarrow \quad h = \frac{2 \tan \theta}{\cos \theta}
$$

Differentiate w.r.t. the variable you are directly measuring (not the variable you are trying to calculate):

(A) $(\sec^2 \theta) \frac{d\theta}{dh} = \frac{1}{2}$

(B) $(\sec^2 \theta) \frac{d\theta}{dt} = \frac{1}{2} \frac{dh}{dt}$

(C) $(\sec^2 \theta) \frac{d\theta}{dt} = \frac{1}{2} \frac{dh}{dt}$

(D) #

Solve for $dh$ to get the differential form

$$
\frac{2}{\cos^2 \theta} d\theta = dh \quad \Rightarrow \quad dh = \frac{2}{\cos^2 \theta} d\theta
$$

Which inequality states (with differential approximation) that:

His height estimate must be accurate to within .01 of a mile.

(A) $dh \leq 0.01$ (B) $\frac{d\theta}{dt} < 0.01$ (C) $d\theta \leq 0.01$ (D) $\frac{d\theta}{dt} \leq 0.01$ (E) #

Thus

$$
\frac{2}{\cos^2 \theta} \frac{d\theta}{dt} < 0.01 \quad \Rightarrow \quad dh = \frac{2}{\cos^2 \theta} \frac{d\theta}{dt}
$$

To get the maximum allowed error in $d\theta$, solve for $d\theta$.

(A) $d\theta < \frac{0.01}{\cos^2 \theta}$

(B) $d\theta < \frac{0.02}{\cos^2 \theta}$

(C) $d\theta < \frac{(0.01)\cos^2 \theta}{2} $$$\quad$$ (D) $d\theta < \frac{\cos^2 \theta}{0.02}$ (E) #

### Parametric curves

Instead describing a curve with a **cartesian** equation such as $y = x^2$ (a parabola) or $x^2 + y^2 = 1$ (unit circle), we may define it with **parametric equations** which write $x$, $y$ as functions of another variable, the **parameter**, usually $t$, which ranges over some interval.

The two parametric equations $x = f(t)$, $y = g(t)$ for $t \in [a, b]$ can be written as single vector equation $(x, y) = (f(t), g(t))$ for $t \in [a, b]$.

- Find parametric equations for the unit circle $x^2 + y^2 = 1$. From a point on the unit circle, the line to the origin forms an angle $t \in [0, 2\pi]$ with the positive x-axis. The point’s coordinates are $(\cos t, \sin t)$. Hence $x = \cos t$, $y = \sin t$, $t \in [0, 2\pi]$ are the parametric equations.

- The graph of $x = \cos t$, $y = 2 \sin t$, $t \in [0, 2\pi]$ is the unit circle graph $(x, y) = (\cos t, \sin t)$ with its $y$-coordinate doubled to $(x, y) = (\cos t, 2 \sin t)$. The graph is the unit circle stretched vertically by a factor of 2 away from the x-axis. This is a vertical ellipse. Find cartesian equations.

$x = \cos t$, $y = 2 \sin t$, $t \in [0, 2\pi]$. Solve for the cos and sin. $\cos t = x$, $\sin t = \frac{y}{2}$. The equation between sin and cos is $\cos^2 t + \sin^2 t = 1$. Replace $\cos t$, $\sin t$ with $x$, $\frac{y}{2}$ to get $x^2 + \frac{y^2}{4} = 1$. This is the desired cartesian equation.

- Find cartesian equations:

(A) $x^2 + \frac{y^2}{4} = 1$

(B) $x^2 + \frac{y^2}{2} = 1$

(C) $\frac{x^2}{2} + \frac{y^2}{3} = 1$

(D) $x^2 + \frac{y^2}{2} = 1$

(E) #

The graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is an ellipse.

The horizontal axis is the $x$-axis interval $[-a, a]$.

The vertical axis is the $y$-axis interval $[-b, b]$.

Matching $\cos^2 t + \sin^2 t = 1$ with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ gives $\frac{x}{a} = \cos t$, $\frac{y}{b} = \sin t$. Solving for $x$ and $y$.

$x = a \cos t$, $y = b \sin t$, $t \in [0, 2\pi]$

These are the desired parametric equations.

- Find parametric equations for the ellipse $\frac{x^2}{3^2} + \frac{y^2}{1^2} = 1$

(A) $x = 9 \sin t$, $y = \cos t$, $t \in [0, 2\pi]$

(B) $x = \sin t$, $y = 9 \cos t$, $t \in [0, 2\pi]$

(C) $x = 9 \cos t$, $y = \sin t$, $t \in [0, 2\pi]$

(D) $x = \cos t$, $y = 9 \sin t$, $t \in [0, 2\pi]$

(E) $x = 3 \cos t$, $y = \sin t$, $t \in [0, 2\pi]