Math 241  Lecture 24


- $h(t) = t^3(3t^2 - 5)$ lead term: $3t^5$ no abs. max. or min.
- $h'(t) = 3t^2(6t - 5)$ Multiply out to expand $h(t)$, differentiate, factor
  (A) $3t^2(6t - 5)$  (B) $18t^3$  (C) $3t^5 - 5t^3$  (D) $15t^2(t + 1)(t - 1)$
- Lead term of $h'(t)$ (lead term of $h'$ not of $h$)
  (A) $18t^2$  (B) $18t^3$  (C) $3t^5$  (D) $15t^4$  (E) #
- List the critical points in increasing order: $x =$
  (A) $0, \sqrt{5/3}$  (B) $-\sqrt{5/3}$, $0, \sqrt{5/3}$  (C) $0, 1$  (D) $-1, 0, 1$
- Pattern of signs for $h'(x)$.
  (A) $- - - -$  (B) $+ - - -$  (C) $+ + + -$  (D) $- + + -$  (E) #
- Pattern of increase/decrease for $h(x)$
  (A) $\nearrow\nearrow\nearrow$  (B) $\nearrow\searrow\nearrow$  (C) $\searrow\nearrow\searrow$  (D) $\searrow\searrow\searrow$  (E) #
- $(-\infty, -1]$ is an interval of
  (A) increase  (B) decrease  (C) both  (D) neither
- The (maximal) interval of decrease is
  (A) $[-1, 0]$  (B) $[-1, 1]$  (C) $[0, 1]$  (D) there are two intervals of decrease
  (E) there is no interval of decrease
- $[1, \infty)$ is an interval of
  (A) increase  (B) decrease  (C) both  (D) neither

Classify the critical points (write in the form $f(a) = c$...).
- $h(-1) = 2$ is a (give the most specific answer)
  (A) loc. max.  (B) loc. min.  (C) crit. pt.  (D) #
- $h(0) = 0$ is a (give the most specific answer)
  (A) loc. max.  (B) loc. min.  (C) crit. pt.  (D) #
- $h(1) = -2$ is a (give the most specific answer)
  (A) loc. max.  (B) loc. min.  (C) crit. pt.  (D) #
- Plot the three critical points. Draw their tangents with short vertical or horizontal line segments. Draw the graph.

- $g(x) = 3x^{1/3}(x^{1/3} - 2)$
- $g'(x) = \frac{2}{x^{2/3}}(x^{1/3} - 1)$ Lead term for $g'$: $\frac{2}{x^{1/3}}$
- Critical points in increasing order: $x = 0, 1$.
- Pattern of signs for $g'(x)$. Note: $x^{2/3} = (x^{1/3})^2$
  (A) $- - -$  (B) $- + +$  (C) $+ + -$  (D) $- - -$  (E) $+ +$
- Pattern of increase/decrease for $g(x)$
  (A) $\searrow\nearrow\nearrow$  (B) $\searrow\searrow\nearrow$  (C) $\nearrow\nearrow\nearrow$  (D) $\nearrow\searrow\nearrow$  (E) $\nearrow\nearrow$
- The (maximal) interval of decrease is
  (A) $(-\infty, 0]$  (B) $(-\infty, 1]$  (C) $[0, 1]$  (D) there are two intervals of decrease
  (E) there is no interval of decrease
- $[1, \infty)$ is an interval of
  (A) increase  (B) decrease  (C) both  (D) neither

Classify the critical points (write in the form $f(a) = c$...).
- $g(0) = 0$ is a (give the most specific answer)
  (A) loc. max.  (B) loc. min.  (C) crit. pt.  (D) #
- $g(1) = -3$ is a (give the most specific answer)
  (A) loc. max.  (B) loc. abs. max.  (C) loc. min.  (D) loc. abs. min
- Plot the two critical points. Draw their tangents with short vertical or horizontal line segments. Draw the graph.
With closed intervals, you can classify the critical points without patterns of signs or intervals of increase/decrease. You can determine their classification just from the plot of their values (include the endpoints).

- The domain is \([-2, 2]\).
- The critical points are \(x = -2, -1, 0, 1, 2\).

The values at these points are

\[
\begin{align*}
f(-2) &= 0, \quad \text{endpoint} \\
f(-1) &= -2, \quad f'(-1) = 0 \\
f(0) &= 0, \quad f'(0) = 0 \\
f(1) &= 2, \quad f'(1) = d.n.e. \\
f(2) &= 0, \quad \text{endpoint}
\end{align*}
\]

Plot these five points. For the three inner critical points, draw their tangents with short vertical or horizontal lines.

- \(f(-2) = 0\) is
  (A) end. max.  (B) end. min.  (C) crit. pt.  (D) end. abs. min.

- \(f(-1) = -2\) is
  (A) loc. max.  (B) loc. abs. min.  (C) loc. min.  (D) crit. pt.

- \(f(0) = 0\) is
  (A) loc. max.  (B) loc. min.  (C) crit. pt.  (D) loc. abs. max.

- The absolute max is at \(x =\)
  (A) -2  (B) -1  (C) 0  (D) 1  (E) 2