**Definite integral**

**Definition** \( \int_a^b f(x) \, dx \), the definite integral of \( f(x) \) from \( a \) to \( b \), (here \( a < b \)) is the signed area between the \( x \)-axis and \( f(x) \) and between the points \( a \) and \( b \). \( a \) is the lower limit, \( b \) is the upper limit, \( x \) is the variable of integration. By signed area, we mean that area below the \( x \)-axis is negative.

Integrating in the backward, right-to-left direction changes the sign. \( \int_0^b f(x) \, dx = -\int_a^b f(x) \, dx \)

1. \( \int_0^1 1 \, dx = 2 \)
2. \( \int_0^1 (-1) \, dx = -2 \)
3. \( \int_0^1 2 \, dx = 2 \)

**Summation (sigma) notation**

The Greek letter \( \Sigma \) sigma designates summation.

\[ \sum_{i=1}^{m} a_i = a_n + a_{n-1} + a_{n-2} + \ldots + a_{m-1} + a_m \]

**i** is the index, **n** is the lower bound, **m** is the upper bound.

- \( \sum_{i=1}^{4} \frac{1}{2} x^i = \frac{1}{2} x^1 + \frac{1}{2} x^2 + \frac{1}{3} x^3 + \frac{1}{4} x^4 \)
- \( \sum_{k=0}^{3} (-1)^k \frac{1}{2} x = (-1)^0 \frac{1}{2} x + (-1)^1 \frac{1}{2} x + (-1)^2 \frac{1}{2} x + (-1)^3 \frac{1}{2} x \)

Starting with \( k = 1 \):

- \( k : 1, 2, 3, 4, \ldots \)
- \( 2k : 2, 4, 6, 8, \ldots \)
- \( 2k-1 : 1, 3, 5, 7, \ldots \)
- \( k^2 : 1, 4, 9, 16, \ldots \)
- \( 2k^2 : 2, 4, 8, 16, \ldots \)

\((-1)^k : -1, 1, -1, 1, \ldots \)

- \((-1)^{k+1} : 1, -1, 1, -1, \ldots \)

**Write in summation (sigma) notation with index** \( k \) **starting at 1**: \( 1 + 3 + 5 + 7 + 9 \)

\[ \sum_{k=1}^{5} (2k-1) \]

**Write in summation (sigma) notation with index** \( k \) **starting at 1**:

\[ 2 + 2^2 x + 2^3 x^2 + 2^4 x^3 + 2^5 x^4 + \ldots \]

\[ = 2^1 x^0 + 2^2 x^1 + 2^3 x^2 + 2^4 x^3 + 2^5 x^4 + \ldots \]

\[ \sum_{i=0}^{\infty} 2^{i+1} x^i = \sum_{k=1}^{\infty} 2^k x^{k-1} \]

**Write in summation (sigma) notation**.

\[ 2x + 3x^2 + 4x^3 + 5x^4 + \ldots \]

- \( \sum_{i=0}^{\infty} ix^{i+1} \)
- \( \sum_{i=0}^{\infty} (i + 1)x^i \)
- \( \sum_{i=0}^{\infty} i x^{i+1} \)
- \( \sum_{i=0}^{\infty} (i + 1)x^i \)
- \( \sum_{i=1}^{\infty} i x^{i+1} \)
- \( \sum_{i=1}^{\infty} (i + 1)x^i \)

**Write in summation (sigma) notation**.

\[ 1 - x + x^2 - x^3 + x^4 + \ldots \]

- \( \sum_{i=0}^{\infty} (-1)^i x^i \)
- \( \sum_{i=0}^{\infty} (-1)^{i+1} x^i \)
- \( \sum_{i=0}^{\infty} (-1)^i x^{i+1} \)
- \( \sum_{i=0}^{\infty} (-1)^{i+1} x^{i+1} \)

**Write in summation (sigma) notation**.

The \( x \) in \( \int_a^b f(x) \, dx \) is the variable of integration, it is the name of the variable for the horizontal axis which could also be any other variable, e.g., \( t \) or \( s \). Since the name we give to the horizontal axis doesn’t affect the size of the region, \( \int_a^b f(x) \, dx = \int_a^b f(t) \, dt = \int_a^b f(s) \, ds \)

For this reason, the variable of integration is called a dummy variable.