Definite integral

**DEFINITION** \( \int_{a}^{b} f(x) \, dx \), the definite integral of \( f(x) \) from \( a \) to \( b \), (here \( a < b \)) is the signed area between the \( x \)-axis and \( f(x) \) - and between the points \( a \) and \( b \). \( a \) is the lower limit, \( b \) is the upper limit, \( x \) is the variable of integration. By signed area, we mean that area below the \( x \)-axis is negative.

Integrating in the backward, right-to-left direction changes the sign. \( \int_{b}^{a} f(x) \, dx = -\int_{a}^{b} f(x) \, dx \)
\[ \int_0^2 1 \, dx = 2 \]
\[ \int_0^2 (-1) \, dx = -2 \]
\[ \int_0^2 1 \, dx = -2 \]
\[ \int_0^2 (-1) \, dx = 2 \]
1. \( \int_0^1 x \, dx = ? \)

2. \( \int_{-1}^0 x \, dx = ? \)

3. \( \int_{-1}^1 x \, dx = ? \)

4. \( \int_1^0 x \, dx = ? \)

5. \( \int_{-1}^{-1} x \, dx = ? \)
The $x$ in $\int_{b}^{a} f(x) \, dx$ is the **variable of integration**, it is the name of the variable for the horizontal axis which could also be any other variable, e.g., $t$ or $s$. Since the name we give to the horizontal axis doesn’t affect the size of the region, 

$$\int_{b}^{a} f(x) \, dx = \int_{a}^{b} f(t) \, dt = \int_{a}^{b} f(s) \, ds$$

For this reason, the variable of integration is called a **dummy variable**.
**Summation (sigma) notation**

The greek letter $\Sigma$ sigma designates summation.

$$\sum_{i=n}^{m} a_i = a_n + a_{n+1} + a_{n+2} + \ldots + a_{m-1} + a_m$$

$i$ is the **index**, $n$ is the **lower bound**, $m$ is the **upper bound**.

- $\sum_{i=1}^{4} \frac{1}{i} x^i = \frac{1}{1} x^1 + \frac{1}{2} x^2 + \frac{1}{3} x^3 + \frac{1}{4} x^4$
- $\sum_{k=0}^{3} (-1)^k \frac{1}{2^k} = (-1)^0 \frac{1}{2^0} + (-1)^1 \frac{1}{2^1} + (-1)^2 \frac{1}{2^2} + (-1)^3 \frac{1}{2^3}$

$k : 1, 2, 3, 4, \ldots$  
Starting with $k = 1$:

$2k : 2, 4, 6, 8, \ldots$  
$2k - 1 : 1, 3, 5, 7, \ldots$

$k^2 : 1, 4, 9, 16, \ldots$

$2^k : 2, 4, 8, 16, \ldots$

$(-1)^k : -1, 1, -1, 1, \ldots$  
$(-1)^{k+1} : 1, -1, 1, -1, \ldots$

- Write in summation (sigma) notation with index $k$ starting at 1:  
  $1 + 3 + 5 + 7 + 9$
  $$\sum_{k=1}^{5} (2k - 1)$$

- Write in summation (sigma) notation with index $k$ starting at 1:  
  $2 + 2^2 x + 2^3 x^2 + 2^4 x^3 + 2^5 x^4 + \ldots$
  
  $= 2^1 x^0 + 2^2 x^1 + 2^3 x^2 + 2^4 x^3 + 2^5 x^4 + \ldots$

  $$\sum_{i=0}^{\infty} 2^{i+1} x^i = \sum_{k=1}^{\infty} 2^k x^{k-1} \quad \leftarrow$$
Write in summation (sigma) notation.
\[2x + 3x^2 + 4x^3 + 5x^4 + \ldots\]
(A) \(\sum_{i=0}^{\infty} ix^{i+1}\)
(B) \(\sum_{i=0}^{\infty} (i + 1)x^i\)
(C) \(\sum_{i=1}^{\infty} ix^{i+1}\)
(D) \(\sum_{i=1}^{\infty} (i + 1)x^i\)
(E) #

Write in summation (sigma) notation.
\[1 - x + x^2 - x^3 + x^4 + \ldots\]
(A) \(\sum_{i=0}^{\infty} (-1)^i x^i\)
(B) \(\sum_{i=0}^{\infty} (-1)^{i+1} x^i\)
(C) \(\sum_{i=0}^{\infty} (-1)^i x^{i+1}\)
(D) \(\sum_{i=0}^{\infty} (-1)^{i+1} x^{i+1}\) (E) #