Exam 2, Monday, covers Lectures 14-24. Consists of homework/classwork problems like the following. Understanding isn’t enough, you must be proficient enough to complete this in 50 minutes.

12(___/4) means a 4-point problem covered in Lecture 12.

Hw 13 1(___/10) \( y = \sqrt{1 + 2t} \)  (a) \( \frac{dy}{dt} \) (b) \( \frac{d^2y}{dt^2} \)
Ans. (a) \( \frac{dy}{dt} = \frac{1}{\sqrt{1 + 2t}} \)
Ans. (b) \( \frac{d^2y}{dt^2} = \frac{1}{2} \left(1 + 2t\right)^{-3/2} \)

Hw 14 2(___/10) A rock is thrown upward. Its height at time \( t \) is \( z = 8t - t^2 \).
What is its acceleration at time \( t \)? Ans. \(-2\)
What is the rock’s maximum height? Ans. 16

Hw 14 3(___/8) The picture shows an object’s velocity \( v \) on the \( y \)-axis at time \( t \).

On what interval(s) of time does the object move upward? Ans. \([2, 8]\)
On what interval(s) of time is the velocity decreasing? Ans. \([6, 9]\)
On what interval(s) of time is it moving at its greatest speed? Ans. \([4, 6]\)
On what interval (of more than one point) of time is the object motionless? Ans. \([0, 2]\)

Hw 15 4(___/8) \( y = \left(2\sqrt{x} - \frac{1}{x}\right)^{-2} \), \( \frac{dy}{dx} = ? \)
Ans. \(-2\left(2\sqrt{x} - \frac{1}{x}\right)^{-3}\left(\frac{1}{2\sqrt{x}} - \frac{1}{x^2}\right)\)

Hw 16 5(___/8) \( r = -\sqrt{\cos(\theta^3)} \), \( \frac{dr}{d\theta} = ? \)
Ans. \( \frac{-1}{2\sqrt{\cos(\theta^2)}} \left(-\sin(\theta^2)\right)(2\theta) = \frac{\theta}{\sqrt{\cos(\theta^2)}} \sin(\theta^2) \)

Hw 17 6(___/8) \( x = t^2 \), \( y = t^2 \) . Find the equation for the line tangent to this curve when \( t = 1 \).
Ans. \( \frac{dy}{dx} = -2t^{-3} \), \( \frac{dy}{dt} = 2t \), \( \frac{dy}{dx} = -t^4 \) when \( t = 1 \), \( x = 1 \), \( y = 1 \), \( \frac{dy}{dx} = -1 \)
\( -y_0 = m(x - x_0) \), \( y - 1 = -(x - 1) \), \( y = -x + 1 + 1 \), \( y = -x + 2 \)

Hw 18 8(___/8) A rectangle of varying width \( x \) and height \( y \) is inscribed in a circle of diameter 2.
(a) Write an equation which relates \( \frac{dy}{dx} \) and \( \frac{dx}{dt} \).
(b) Find \( \frac{dx}{dt} \) when \( x = 1 \) if the rate of decrease of \( y \) is 3.
Ans. (a) \( x^2 + y^2 = 2^2 \), \( 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \), \( x\frac{dx}{dt} = -y\frac{dy}{dt} \).
Ans. (b) \( \frac{dy}{dt} = -3 \), \( \frac{dx}{dt} = \frac{-y}{x} \frac{dy}{dt} = \frac{-y}{x}(-3) = \frac{3y}{x} \)

When \( x = 1 \), \( 1^2 + y^2 = \left(\sqrt{5}\right)^2 \), \( y^2 = 5 - 1 = 4 \), \( y = 2 \),
\( \frac{dx}{dt} \bigg|_{x=1} = \frac{3y}{x} \bigg|_{x=1,y=2} = \frac{3(2)}{1} = 6 \)
Pedestrian $A$ walks toward an intersection $I$ from the north at 2 meters/second. Pedestrian $B$ walks westward away from the intersection at 1 meter/second. What is the rate of change of the angle $\theta$ between the ray from $B$ to the intersection and the ray from $B$ to $A$ (the angle $\angle IBA$), when $A$ is 10 meters from the intersection and $B$ is 10 meters away? This is basically homework 19, problem 36(5). Include units.

欲求 $\frac{d\theta}{dt}|_{x=10, y=10}$

已知 $\frac{dx}{dt} = -1$, $\frac{dy}{dt} = -2$, $\tan \theta = \frac{y}{x}$

等式 $\tan \theta = \frac{y}{x}$

微分 $\sec^2 \theta \frac{d\theta}{dt} = \frac{(dy/dt)x - y(dx/dt)}{x^2}$

$$\frac{1}{\cos^2 \theta} \frac{d\theta}{dt} = \frac{(-2)x - y(1)}{x^2}, \quad \frac{d\theta}{dt} = \cos^2 \theta \frac{-2x - y}{x^2}$$

答案，当 $x=1, y=1$, 需要 $\cos \theta$. 由勾股定理，斜边为 $\sqrt{2}$. 因此 $\cos \theta = \frac{1}{\sqrt{2}}$.

$$\frac{d\theta}{dt}|_{x=10, y=10} = \left(\frac{1}{\sqrt{2}}\right)^2 \frac{-2(10) - (10)}{(10)^2} = -\frac{3}{20} \text{ rad./sec.}$$

SX 10(____/8) 你必须测量一个罐子（右圆柱形）的体积在0.1立方米。你需要准确地测量出你的圆的半径，如果半径的估计是5英尺，那么实际的高度（这里是一个常数）是已知的为5英尺。写您的答案没有小数。

$v = \pi r^2 h$, $dv = 10 \pi r dr$, $dv/0.1 \Rightarrow 10 \pi (5)dr \leq 0.1 \Rightarrow dr \leq 0.1/50\pi = 1/500\pi \text{ feet}$. 

SX 23 10(____/14) 列出，按照递增的顺序，增加或减少的区间。列出并分类临界点。将形式为 $f(1) = 2$ abs loc max or $f(2) = f(4) = -1$ endpt. min. $f(x) = \sqrt{x^2 - 2x - 3} = \sqrt{(x-3)(x+1)}$

答案，主导项为 $f_0(x) = \sqrt{x}$. 方程 $f$ 是 $|x|$. 域 $(-\infty, -1] \cup [3, \infty)$

Der: $f'(x) = \frac{2x - 2}{2\sqrt{x^2 - 2x - 3}} = \frac{2(x-1)}{(x-3)(x+1)}$

主导项为 $f'_0(x) = \frac{2x}{\sqrt{x^2}} = \frac{2x}{|x|} = -2$ if $x \in f(-\infty, -1]$,

等于 $2$ if $x \in [3, \infty)$

Hence $f' < 0$ if $x \in (-\infty, 1]$. $f' > 0$ if $x \in [3, \infty)$

Crit. pts.

$x = -1(\text{end}, f'(dne), 3(\text{end}, f'(dne)$ Note, $f' = 0$ at 1 but 1 is not in the domain.

$(-\infty, -1]$, $f' < 0$, decreasing

$[3, \infty)$, $f' > 0$, increasing

$f(-1) = 0, f(3) = 0$

Classify the critical points and their values.

$f(-3) = f(1) = 0$ abs. and end. min.