Math 241   Exam 2 Review Lecture   Lectures 14-24

Hw 7 means problem is covered in Lecture/Homework 7.

Hw 13     1(/10)  
\[ z = \frac{3x-1}{x-1} \]
(a) Find \( \frac{dz}{dx} \) (b) Find \( \frac{d^2z}{dx^2} \)
Ans. (a) \( \frac{3(x-1)-(3x-1)}{(x-1)^2} = \frac{-2}{(x-1)^2} \)
Ans. (b) \( -2((x-1)^{-2})' = 4(x-1)^{-3} \)

Hw 14     2(/10)  A rock is thrown upward. Its height at time \( t \) is \( y = 8t - 2t^2 \). Remember the units.
What is its acceleration at time \( t \)?  Ans. -4 ft./sec./sec.
What is the rock’s maximum height?  Ans. 8 ft.

Hw 14     3(/8)  The picture shows an object’s velocity \( v \) on the \( y \)-axis at time \( t \).

On what interval(s) of time does the object move upward?
Ans. [8, 9]
On what interval(s) of time is the velocity decreasing?
Ans. [0, 2]
On what interval(s) of time is it moving at its greatest speed?
Ans. [2, 3]
On what interval (of more than one point) of time is the object motionless?
Ans. [6, 8]

Hw 15     4(/8)  \( y = (\frac{x^2}{2} - 2\sqrt{x})^{-5} \), \( \frac{dy}{dx} = ? \)
Ans. \( -5(\frac{x^2}{2} - 1)^{-6}(x + \frac{1}{\sqrt{x}}) \)

Hw 16     5(/8)  \( r = \frac{1}{\theta} \sin^3(\theta^2) \), \( \frac{dr}{d\theta} = ? \)
Ans. \( \frac{1}{6}3\sin^2(\theta^2)\cos(\theta^2)2\theta = \theta \sin^2\theta^2 \cos \theta^2 \)

Hw 21     6(/12)  \( x = 1/t \), \( y = t^3 \). Find the equation for the line tangent to this curve when \( t = 1 \).
Ans. \( dx/dt = -1/t^2 \), \( dy/dt = 3t^2 \), \( dy/dx = -3t^4 \) when \( dy/dx |_{t=1,y=1} = -3 \)
\( y - y_0 = m(x - x_0) \) y = 1 = (-3)(x - 1),
\( y = -3x + 3 + 1, \ y = -3x + 4 \)

Hw 17     7(/8)  \( 2y - xy = 1 \)
(a) Find \( \frac{dy}{dx} \) (b) Find \( \frac{d^2y}{dx^2} \)
Ans. (a) \( 2y' - (x + y'y') = 0 \), \( y' = \frac{y}{2-x} \)
Ans. (b) \( y'' = \left(\frac{y}{2-x}\right)' = \frac{y'(2-x) - y(1)}{(2-x)^2} = \frac{\frac{y}{2-x}(2-x) + y}{(2-x)^2} = \frac{2y}{(2-x)^2} \)

Hw 18     8(/8)  The area of a rectangle with base \( x \) and height \( y \) is 4 square feet. If the base and height vary with time \( t \) write an equation which relates \( \frac{dy}{dt} \) and \( \frac{dx}{dt} \).
Ans. \( xy = 4 \), the equation is \( \frac{dx}{dt}y + x \frac{dy}{dt} = 0 \) or \( y \frac{dx}{dt} = -x \frac{dy}{dt} \)

Hw 19     9(/20)  A point moves along the curve \( y = x^3 \). The \( x \)-coordinate increases at 3 feet/sec. How fast is the angle of elevation of the point as seen from the origin changing when \( x = 1 \) feet? This is the angle between the ray from the origin to the point and the positive \( x \)-axis.
Include units.
Want \( \frac{d\theta}{dt} \) | \( x=1 \)
Given \( y = x^3 \), \( \frac{dx}{dt} = 10 \), \( \tan \theta = \frac{x^3}{x} = x^2 \)
Eq. Need equation between \( \theta \) and \( x \).
\( \tan \theta = x^2 \)
Diff. \( \sec^2 \theta \frac{d\theta}{dt} = 2x \frac{dx}{dt} \cdot \frac{1}{\cos^2 \theta} \frac{d\theta}{dt} = 2x \frac{dx}{dt} \cdot \frac{d\theta}{dt} = 2x \cos^2 \theta \frac{dx}{dt} = 2x \cos^2 \theta(3) = 6x \cos^2 \theta \)
Ans. When \( x = 1 \), need cos \( \theta \). By the Pythagorean Theorem, the hypotenuse is \( \sqrt{x^2 + x^2} \). When \( x = 1 \) this is \( \sqrt{1^2 + 1^2} = \sqrt{2} \). Hence \( \cos \theta = \frac{x}{\sqrt{x^2 + x^2}} = \frac{1}{\sqrt{2}} \).
\( \frac{d\theta}{dt} |_{x=1} = 6x \cos^2 \theta = \frac{6(1)}{(\sqrt{2})^2} = 3 \) rad./sec.
Hw 20  10(___/8) You must measure the volume of a sphere to within .1 cubic foot. How accurate must you measure the radius if your radius estimate is 10 feet? Write your answer without decimals.

Ans. \[ V = \frac{4\pi r^3}{3}, \quad dV = 4\pi r^2 dr, \]
\[ dV \leq .1 \Rightarrow 4\pi (10)^2 dr < .1 \Rightarrow dr \leq \frac{1}{400\pi} = \frac{1}{4000\pi} \text{ feet} \]

Hw 23 24  10(___/14) Classify the critical points and draw the graph. Classify as abs/loc/end max/min/crit. Write in the form abs loc max or endpt. min.

\[ f(x) = \sqrt{3 - 2x - x^2} = \sqrt{-(x - 1)(x + 3)} \]

Ans. Domain \([-3, 1]\]

Diff. \[ f'(x) = \frac{-2 - 2x}{2\sqrt{3 - 2x - x^2}} = \frac{-(x + 1)}{\sqrt{-(x - 1)(x + 3)}} \]

Crit. pts. \[ f' = 0, \quad f' \text{ undef., endpoints.} \]
\[ x = -3 (\text{end, } f' \text{ dne}), -1 (f'' = 0), 1 (\text{end, } f' \text{ dne}) \]

Values \[ f(-3) = 0, \quad f(-1) = 2, \quad f(1) = 0 \]

Classify the extremes. Classify as abs/loc/end/crit. Write in the form \( f(1) = 2 \text{ abs loc max or loc min, etc.} \)

\[ f(-1) = 2 \text{ abs. loc. max.} \]
\[ f(-3) = f(1) = 0 \text{ abs. endp. min.} \]

Graph Draw the graph, an upper semicircle with center ((-1,0) and radius 2. Draw the three tangents with short vertical or horizontal line segments.