Exam 3, Monday, covers Lectures 25-34. Consists of homework/classwork problems like the following. Understanding isn’t enough, you must be proficient enough to complete this in 50 minutes.

**HW 25** 1(___/12) Let \( s(t) \), \( v(t) \), \( a(t) \) be the position, velocity and acceleration of a point on the \( y \)-axis at time \( t \). Find the position \( s(t) \) at any time \( t \) given that:
\[
 a(t) = \sin t, \quad v(0) = 0, \quad s(0) = 0.
\]

**Ans.** \( v(t) = -\cos t + C \)
\( v(0) = 0 \)
\(-1 + C = 0 \)
\( C = 1 \)
\( v(t) = -\cos t + 1 \)
\( s'(t) = v(t) = -\cos t + 1 \)
\( s(t) = -\sin t + t + D \)
\( s(0) = 0 \)
\(-0 + 0 + D = 0 \)
\( D = 0 \)
\( s(t) = -\sin t + t \)

**HW 26-28** 3(___/20) Graph. On the graph, give the coordinates of critical points and inflection points. Classify as specifically as possible: loc. abs. max/min, loc. max/min., crit. pt., infl. pt. When calculating the second derivative, factor the numerator as early as possible to avoid a mess.
\[
y = f(x) = \frac{x}{x^2 + 1}
\]

**Ans.** lead term \( 1/x \)
\[
y' = \frac{(x^2 + 1) - x(2x)}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2} \quad \text{lead term } -1/x^2
\]
\[
y'' = \frac{-2x(x^2 + 1)^2 - (1-x^2)(2(x^2 + 1)(2x))}{(x^2 + 3)^4}
\]
\[
= \frac{-2x(x^2 + 1)(x^2 + 1) + (1-x^2)2}{(x^2 + 3)^4}
\]
\[
= \frac{-2x(x^2 + 1)(-x^2 + 3)}{(x^2 + 1)^4} = \frac{2x(3-x^2)}{(x^2 + 1)^3} \quad \text{lead term } -2/x^3
\]

Inf. pts. \( f(-\sqrt{3}) = -\sqrt{3}/4, \quad f(0) = 0, \quad f(1) = \sqrt{3}/4 \).
Intervals: \( (-\infty, -\sqrt{3}) \) conv. up., \( (-\sqrt{3}, 0) \) conv. down, \( (0, \sqrt{3}) \) conv. down, \( (\sqrt{3}, \infty) \) conv. up.
Crt. pts: \( x = 0, f(0) = 1/3 \) loc. abs. min.

**HW 29-32** 5(___/18) Problem 5, Page 226. You have a 8 inch by 15 inch rectangle of cardboard. You cut out congruent squares from each corner and then fold up the sides to form a box with open top. Find the size of the squares which should be cut out to give the largest possible volume of the a box. Instead of finding the size, you need only find the equation whose root is the desired answer.

**Ans:**

Given(1):
\[
V = x(15 - 2x)(8 - 2x)
\]

One variable(2):
\[
V = 4x^3 - 46x^2 + 120x
\]

Critical points(2):
\[
V' = 12x^2 - 92x + 120
\]
\[
= 0 \iff 12x^2 - 92x + 120 = 0 \iff x = 5/3, 6 \]

is too big, so the answer is \( x = 5/3 \) but only the equation is required.
What is the largest possible area of an isosceles triangle inscribed in the unit circle? Hint: Picture the isosceles triangle as symmetric across the y-axis. For the largest area, the horizontal base will be below the x-axis. From the origin, draw three radii (each has length one) to the three corners of the triangle. Let $x$ be the distance between the origin and the base of the triangle. The area can be written in terms of $x$.

**Ans.**

1. **Picture:** Draw the picture and indicate the variables.

2. **Given:** List the given facts.
   - $x^2 + y^2 = 1$
   - $A = \frac{1}{2}(2y)(1 + x)$
3. **One variable:**
   - $y = \sqrt{1 - x^2}$
   - $A = y(1 + x) = \sqrt{1 - x^2} (1 + x)$
4. **Critical points:**
   - $A' = \frac{1}{2\sqrt{1-x^2}}(-2x)(1 + x) + \sqrt{1-x^2} (1)
     \begin{align*}
     &= \frac{-x - x^2}{\sqrt{1-x^2}} + \frac{1 - x^2}{\sqrt{1-x^2}} \\
     &= \frac{1 - x - 2x^2}{\sqrt{1-x^2}} \\
     A' &= 0 \text{ iff } 2x^2 + x - 1 = 0 \text{ iff } (2x - 1)(x + 1) = 0 \text{ iff }
    \begin{align*}
    x &= -1, \ 1/2 \text{ iff } x = 1/2
    \end{align*}
\end{align*}$
   - $A = \sqrt{1 - x^2} (1 + x) = \sqrt{3/4} (1 + 1/2)
     = (\sqrt{3}/2)(3/2) = 3\sqrt{3}/4$

5. **Answer:** The largest area is $3\sqrt{3}/4$. (No units given.)

**Hw 33-34**

Solve the differential equation and its initial value.

1. **Per 2(12)**
   \[
   \frac{dy}{dx} = \sqrt{x}, \ y(4) = \frac{1}{3}
   \]
2. **Ans.**
   \[
   y = \frac{3^{3/2}}{3^{3/2}} + C = \frac{3}{3^{3/2}} + C
   \]
   \[
   \frac{2}{3}4^{3/2} + C = \frac{1}{3}
   \]
   \[
   C = \frac{1}{3} - \frac{16}{3} = \frac{-15}{3} = -5
   \]

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**Math 241 Practice Exam 3**

Complete both 2-sided pages. Simplify answers, put in the space provided. Scratch paper isn’t graded. No decimals.