Hw 25

1(1/12) \( a(t) = -2t \), \( v(0) = 1 \), \( s(0) = 3 \).

Ans. \( v'(t) = a(t) = -2t \)
\[ v(t) = -t^2 + C \]
\[ v(0) = 1 \]
\[ -0^2 + C = 1 \]
\[ C = 1 \]
\[ v(t) = -t^2 + 1 \]
\[ s'(t) = v(t) = -t^2 + 1 \]
\[ s(t) = -\frac{1}{3}t^3 + t + D \]
\[ s(0) = 3 \]
\[ -2(0)^3 + 0 + D = 3 \]
\[ D = 3 \]
\[ s(t) = -\frac{1}{3}t^3 + t + 3 \]
Graph. On the graph, give the coordinates of critical points (classify) and inflection points.

\[ y = f(x) = \frac{1}{x^2 + 3} \]

Ans.

\( y \)-int: \( y = 1/3 \), \( x \)-int. none, lead term \( 1/x^2 \) hor. asymp. \( y = 0 \)

\( y' = \frac{-2x}{(x^2 + 3)^2} \) lead term \(-1/x^3\)

Crit. pts: \( x = 0, f(0) = 1/3 \) loc. abs. max.

Intervals of incr/decr: \( \uparrow (-\infty, 0], \downarrow [0, \infty) \)

\( y'' = \frac{-2(x^2 + 3)^2 + 2x[2(x^2 + 3)(2x)]}{(x^2 + 3)^4} = \frac{2(x^2 + 3)[-(x^2 + 3) + 4x]}{(x^2 + 3)^4} \)

\[ = \frac{2(x^2 + 3)(3x^2 - 3)}{(x^2 + 3)^4} = \frac{6(x^2 - 1)}{(x^2 + 3)^3} = \frac{6(x + 1)(x - 1)}{(x^2 + 3)^3} \] lead \( 6/x^4 \)

Infl. pts: \( f(-1) = 1/4, f(1) = 1/4 \).

Intervals: \( \cup (-\infty, -1) \) conv. up., \( \curvearrowleft (-1, 1) \) conv. down, \( \cup (1, \infty) \) conv. up.
$f(-1) = \frac{1}{4}$ infl. pt.

$f(0) = \frac{1}{3}$ loc. abs. max.

$f(1) = \frac{1}{4}$ infl. pt.
Hw 26-28 4(___/20)  \[ y' = f'(x) = x^{-2/3}(x - 1) = \frac{x-1}{x^{2/3}}, \quad y(0) = 0 \]

Ans.
\[ y' = x^{1/3} - x^{-2/3} \quad \text{lead term } x^{1/3} \]

Crit. pts: \( x = 1 \) loc. abs. min., \( x = 0 \) crit. pt. \( f'(0) \) d.n.e.

Intervals of incr/decr: \( \downarrow (-\infty, 1], \quad \uparrow [1, \infty) \)

\[ y'' = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3} = \frac{1}{3x^{5/3}}(x + 2) \quad \text{lead term } \frac{1}{3x^{2/3}} \]

Infl. pts.: \( x = 0, 2 \)

Intervals of concavity: \( \cup (-\infty, -2), \quad \cap (-2, 0), \quad \cup (0, \infty) \)

Graph
Consider the semicircle $y = \sqrt{1 - x^2}$ above the $x$-axis of radius one foot around the origin. Find the width of the largest rectangle which lies inside this semicircle.

**Ans.**

**Picture:**

![Diagram of semicircle and rectangle](image)

**Given:** List the given facts.

- $y = \sqrt{1 - x^2}$
- $A = 2xy$

**One variable:**

- $A = 2x\sqrt{1 - x^2}$

**Domain:** $x \in [0, 1]$
Diff.:

\[ A' = 2\left[ \sqrt{1-x^2} + x \frac{(-2x)}{\sqrt{1-x^2}} \right] = 2 \frac{1-2x^2}{\sqrt{1-x^2}} \]

Critical points:

endpoints, \(x = 0, 1\).

\(A'\) d.n.e. \(x = 0\)

\(A' = 0\) iff \(1-2x^2 = 0\) iff \(x = \frac{1}{\sqrt{2}}\)

Width = \(2x = \frac{2}{\sqrt{2}} = \sqrt{2}\)

Answer: The width is \(\sqrt{2}\) feet

Proof: \(A(0) = A(1) = 0, A(1/\sqrt{2}) > 0\)
Second and harder word problem. 

A wall is 8 feet high and 27 feet from a building. We want the shortest ramp from the ground to the building across the top of the wall. How far from the wall will this beam contact the ground? Omit proof.

Picture:

Given:

\[ z = \sqrt{(x + 27)^2 + y^2} \]

\[ \frac{y}{x + 27} = \frac{8}{x} \]

One variable: domain: \( x \in (0, 8) \)

\[ y = 8 \frac{x + 27}{x} \]
\[ z = \sqrt{(x + 27)^2 + 64\left(\frac{x + 27)^2}{x^2}\right)} \]

\[ = (x + 27)\sqrt{1 + \frac{64}{x^2}} \]

\[ = \frac{x + 27}{x}\sqrt{x^2 + 64} \]

\[ = (1 + \frac{27}{x})\sqrt{x^2 + 64} \]

**Diff.:**

\[ \frac{dz}{dx} = \frac{-27}{x^2}\sqrt{x^2 + 64} + (1 + \frac{27}{x})\frac{2x}{2\sqrt{x^2 + 64}} \]

\[ = \frac{-27(x^2 + 64) + x^2(1 + \frac{27}{x})x}{x^2 \sqrt{x^2 + 64}} \]

\[ = \frac{-27x^2 - (27)(64) + x^3 + 27x^2}{x^2 \sqrt{x^2 + 64}} \]

\[ = \frac{x^3 - (27)(64)}{x^2 \sqrt{x^2 + 64}} \]
Crit. pts.: endpts, none. d.n.e. never on domain.

\[ \frac{dz}{dx} = 0 \text{ iff } x^3 = (27)(64) = (3^3)(4^3) \]

iff \( x = 12 \)

Answer: The distance between the ramp on the ground and the wall is 12 feet.

Hw 33-34
Solve the differential equation and its initial value.

\[ 2\left(\frac{\text{___}}{12}\right) \frac{ds}{dt} = \sin t, \ s(0) = 1 \]

Ans. \( s = -\cos t + C \)

\[ -\cos 0 + C = 1 \]

\( (-1) + C = 1 \)

\( C = 2 \)

\( s = -\cos t + 2 \)