Exam 1, Friday, covers Lectures 1-13. Consists of homework/classwork problems like the following. Understanding isn't enough, you must be proficient enough to complete this in 50 minutes.  

2b(__/4) means a 4-point problem covered in Lecture 2.

1a(__/4) Find the average rate of change of \( g(x) = x^2 + 2x \) over the interval \([-1, 1]\).

1b(__/9) \( y = x^2 + x \). Find the equation for the tangent to the curve at \((1, 2)\). Write equation in the form \( y = ax + b \).

2a(__/4) Find the limit or write “d.n.e.” if the limit does not exist. \( \lim_{x \to 3} \frac{x + 3}{x^2 + 4x + 3} \)

2b(__/5) \( \lim_{x \to -1} \frac{x + 1}{\sqrt{x^2 + 8} - 3} \)

*3a(__/10) Use the sandwich (pinching) theorem to prove: \( \lim_{\theta \to \infty} \frac{\cos \theta}{\theta} = 0 \)

3b(__/5) Find the limit \( \lim_{h \to 0} \frac{\tan(h)}{8h} \). If your answer is not a number, it should be \( \infty \) or \( -\infty \) rather than d.n.e.

*4(__/10) \( f(x) = \frac{1}{x} \), use the definition of the derivative to prove that \( f'(x) = -\frac{1}{x^2} \).

5(__/5) Find the limit. If it does not exist, write \( \infty \) or \( -\infty \) or “d.n.e.”, whichever is most specific. \( \lim_{x \to -2} \sqrt{x + 2} \)

6(__/5) \( \lim_{x \to \infty} \frac{1/(2x + 3)}{1/(3x + 2)} \)

7,8(__/19) Graph \( y = \frac{2x^2}{x^2 - 1} \). Also draw the asymptotes. Label them with their equations. Label the zeros (roots).

8(__2) Classify the state of continuity at \( x = 0 \) (more than one might apply): continuous, removable discontinuity, essential discontinuity, continuous from the left, continuous from the right.

\[ f(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ 1 - x^2 & x < 0 \end{cases} \]

*9(__/10) Prove that the equation \( \cos x = -x \) has a solution in \([-\pi/2, 0]\). Use the Graph Intersection Theorem.

10(__/7) Graph \( y = \sqrt{1 - (x + 1)^2} \).

11(__/7) Find the rate of change of the volume of a ball \( V = \frac{4}{3} \pi r^3 \) with respect to the radius when the radius is \( r = 1 \).

12(__/7) Given the graph of \( f(x) \) below. Draw the graph of \( f'(x) \).

\[ \text{Graph} \]

13a(__/7) \( y = \frac{\sin x}{\cos x} \). Find \( \frac{dy}{dx} \). Find \( \frac{dy}{dx} \big|_{x=0} \).

13b(__/5) \( y = -2 \sin x \). Find \( \frac{d^2y}{dx^2} \).

Answers:
1a (3) \( -1 - (-1) = 2 \)
1b \( y' = 2x + 1, \ y - 2 = 3(x - 1), \ y = 3x - 1 \)
2a -1/2
2b -3
*3a Proof*
3b 1/8
*4 Proof*
5 d.n.e.
6 3/2
7,8 roots: 0, v.a.: \( x = -1, 1 \), h.a.: \( y = 2 \)
8 essential discontinuity, continuous from the right.
*9 Proof*
10 Graph
11 4\pi
12 Graph
13a sec^2x, 1
13b 2sinx

*This practice exam has three proofs. Only one will be on Exam 1.

Complete both pages, both sides