EXACTNESS AND THE NOVIKOV CONJECTURE

ERIK GUENTNER AND JEROME KAMINKER

Abstract. In this note we will study a connection between the conjecture that $C^*_r(\Gamma)$ is exact and the Novikov conjecture for Γ. The main result states that if the inclusion of the reduced C^*-algebra $C^*_r(\Gamma)$ of a discrete group Γ into the uniform Roe algebra of Γ, $UC^*(\Gamma)$, is a nuclear map then Γ is uniformly embeddable in a Hilbert space. By a result of G. Yu, this implies that Γ satisfies the Novikov conjecture. Note that the hypothesis is a slight strengthening of the usual notion of exactness since a group Γ is exact if and only if the inclusion of $C^*_r(\Gamma)$ into $\mathcal{B}(l^2(\Gamma))$ is nuclear.

1. Introduction

Let X be a discrete metric space with metric d. A function f from X to a separable Hilbert space $\mathcal{H}$ is a *uniform embedding* if there exist non-decreasing proper functions $\rho_\pm : [0,\infty) \to [0,\infty)$ such that

$$\rho_-(d(x,y)) \leq \|f(x) - f(y)\| \leq \rho_+(d(x,y)), \quad \text{for all } x,\, y \in X.$$

The Strong Novikov Conjecture states that the assembly map on K-theory,

$$\mu : K_*(B\Gamma) \to K_*(C^*_r(\Gamma)),$$

is rationally injective. Answering a question of Gromov, Yu proved the following theorem [16, 12].

Theorem 1.1. *Let Γ be a finitely presented discrete group. If Γ is uniformly embeddable in a Hilbert space, then Γ satisfies the Strong Novikov conjecture.*

The first author was supported in part by NSF Grant DMS-9706960.

The second author was supported in part by NSF Grant DMS-9706817.

This is currently the weakest general hypothesis implying the Novikov conjecture. It is conceivable, however, that there exist groups which are not uniformly embeddable in a Hilbert space but which nevertheless satisfy the Novikov conjecture. At present there are no such examples known.

From another direction, there is the question of whether all finitely generated discrete groups are exact, [8, 10]. Recall that a discrete group Γ is *exact* if its reduced C^*-algebra, $C_r^*(\Gamma)$, is an exact C^*-algebra. That is, given the exact sequence of C^*-algebras

$$0 \to I \to B \to B/I \to 0$$

the sequence

$$0 \to I \otimes_{min} C_r^*(\Gamma) \to B \otimes_{min} C_r^*(\Gamma) \to (B/I) \otimes_{min} C_r^*(\Gamma) \to 0$$

is also exact.

The Novikov conjecture and Exactness question appear to have little in common other than that they both involve properties which might be possessed by all finitely presented groups. However, there is a link provided by results of Roe-Higson and Yu [7, 16]. Combined these results state that if Γ acts amenably, in the topological sense [2], on a compact space then it is uniformly embeddable in a Hilbert space, and hence satisfies the Novikov conjecture. On the other hand, it is an easy observation that this condition also implies that Γ is exact. Thus, the same hypothesis yields both properties.

We note that Gromov has asserted the existence of finitely presented groups that are not uniformly embeddable [6]. This follows from the construction of a finitely presented group whose Cayley graph contains, in an appropriate sense, a sequence of expanding graphs together with the observation that a sequence of expanding graphs [11], when viewed as a discrete metric space in a natural way, is not uniformly embeddable. On the other hand, Higson has observed that the uniform Roe algebra of such a discrete metric space is, in general, not an exact C^*-algebra. (This also follows from a result of Voiculescu [14].) Based on these remarks it seems likely that

Gromov's examples of non-uniformly embeddable groups will in general fail to be exact.

The purpose of this note is to study the relationship between uniform embeddability and exactness. We state the main result, leaving precise definitions for later in the paper. We need the uniform Roe algebra, $UC^*(\Gamma)$, sometimes called the "rough" algebra, introduced by Roe, [7]. It is isomorphic to the reduced cross product $C(\beta\Gamma) \rtimes_r \Gamma$. The left regular representation provides an inclusion of $C_r^*(\Gamma)$ into $\mathcal{B}(l^2(\Gamma))$, and in fact into $UC^*(\Gamma)$. Recall that a group Γ is exact if this inclusion is a nuclear embedding of $C_r^*(\Gamma)$ into $\mathcal{B}(l^2(\Gamma))$ [15, 8]. We modify this condition by requiring that the inclusion be a nuclear embedding of $C_r^*(\Gamma)$ into $UC^*(\Gamma)$. Note that most classes of groups which are know to be exact, including word hyperbolic groups, discrete subgroups of connected Lie groups, Coxeter groups, etc., actually satisfy this stronger condition.

Theorem 1.2. *Let Γ be a finitely presented group. If the inclusion of the reduced C^*-algebra $C_r^*(\Gamma)$ into the uniform Roe algebra $UC^*(\Gamma)$ is a nuclear map, then Γ is uniformly embeddable in a Hilbert space, and hence satisfies the Novikov conjecture.*

It is natural to consider other refinements of exactness which can be obtained by replacing $UC^*(\Gamma)$ by other subalgebras of $\mathcal{B}(l^2(\Gamma))$. This will be discussed in a future paper.

We would like to thank Nigel Higson and John Roe for useful and enjoyable conversations.

2. Approximate units and negative type functions

In this section we will assemble some of the facts needed for the results in Section 3. In particular, we will establish an analog of a theorem of Akemann-Walter, [1].

A complex-valued function f on the set $X \times X$ is said to be *positive definite* if, for any $n \geq 1$,

$$\sum_{ij} \overline{z}_i f(x_i, x_j) z_j \geq 0, \quad \text{for all } x_1, \ldots, x_n \in X \text{ and } z_1, \ldots, z_n \in \mathbb{C}.$$

A real-valued function h on $X \times X$ is of *negative type* if

(i) $h(x,x) = 0$ for all $x \in X$,
(ii) $h(x,y) = h(y,x)$ for all x, $y \in X$, and
(iii) $\sum_{ij} a_i h(x_i, x_j) a_j \leq 0$, for all $x_1, \ldots, x_n \in X$ and $a_1, \ldots, a_n \in \mathbb{R}$ satisfying $\sum_j a_j = 0$.

It will be convenient to have the following notation. If X is a metric space and A is a subspace, then $C_0(X; A)$ will be the set of functions which tend to zero off of A. That is, $f \in C_0(X; A)$ if for any $\varepsilon > 0$ there is an $R > 0$ such that $|f(x)| < \varepsilon$ if $d(x, A) > R$.

Suppose now that X is a discrete metric space. Consider the ideal $C_0(X \times X; \Delta) \subseteq l^\infty(X \times X)$, where Δ denotes the diagonal of $X \times X$. A net $f_\lambda \in l^\infty(X \times X)$ satisfies $\|f_\lambda f - f\| \to 0$ for all $f \in C_0(X \times X; \Delta)$ if and only if $f_\lambda \to 1$ uniformly on any set of the form $B_\Delta(R) = \{(x,y) : d(x,y) < R\}$. Finally, we say that a complex-valued function f on $X \times X$ is *metrically proper* if it satisfies that for any $C > 0$ there is an $R > 0$ such that $|f(x,y)| > C$ if $d(x, \Delta) > R$.

The following result is a generalization of [1, Theorem 10] from the case of groups to that of equivalence relations.

Theorem 2.1. *Let X be a countable discrete metric space. There exists an approximate unit for $C_0(X \times X; \Delta)$ consisting of positive definite functions if and only if there exists a metrically proper negative type function on $X \times X$.*

Proof. Let ϕ be a metrically proper negative type function on $X \times X$. By a generalization of Schoenberg's Theorem, [4], the function $e^{-t\phi(x,y)}$ is positive definite for any $t \geq 0$. Since ϕ is metrically proper, one has, for any t, $e^{-t\phi(x,y)} \in C_0(X \times X; \Delta)$. On the other hand one also has $\lim_{t \to 0} \|e^{-t\phi(x,y)} - 1\| = 0$ uniformly on $B_\Delta(R)$ for any $R > 0$. Thus, $\Phi_t = e^{-t\phi}$ provides the approximate unit for $C_0(X \times X; \Delta)$ consisting of positive definite functions.

For the converse, let u_λ be an approximate unit consisting of positive definite functions. Since $u_\lambda \to 1$ uniformly on $B_R(\Delta)$, there exists an R and λ_0 so that $u_\lambda > 0$ if $d(x,y) < R$ and $\lambda > \lambda_0$. One may thus adjust the approximate unit so that $u_\lambda(x,x) = 1$ for all $x \in X$. Now, exactly as in [1, Theorem 10] one extracts a sequence u_{λ_i} such that the function $\sum_n Re(1 - u_{\lambda_n})2^n$ converges to the required metrically proper negative type function. □

We next recall the result of Dranishnikov-Gong-Lafforgue-Yu, [5], relating metrically proper negative type functions to uniform embeddings in a Hilbert space.

Theorem 2.2. *A locally finite metric space X is uniformly embeddable in a Hilbert space if and only if there exists a metrically proper negative type function on $X \times X$.*

Combining these two results we obtain

Theorem 2.3. *The following are equivalent for the countable, locally finite, discrete metric space X.*

(i) X is uniformly embeddable in a Hilbert space.
(ii) There is a metrically proper negative type function on $X \times X$.
(iii) There is an approximate unit for $C_0(X \times X; \Delta)$ consisting of positive definite functions.

In Section 3 we will discuss the relation of this to the Haagerup property for the groupoid $\beta\Gamma \rtimes \Gamma$.

3. Exactness

In this section we restrict X to be a finitely presented group with a length function determined by a finite, symmetric set of generators. The length function, l, determines a right invariant metric via $d(s,t) = l(st^{-1})$. The quasi-isometry type of (Γ, d) is independent of the choice of generators. We next recall the definition of the uniform Roe algebra associated to (Γ, d).

Consider the set of $A : \Gamma \times \Gamma \to \mathbb{C}$ satisfying

(i) there exists $M > 0$ such that $|A(s,t)| \leq M$, for all s, $t \in \Gamma$
(ii) there exists $R > 0$ such that $A(s,t) = 0$ if $d(s,t) > R$

Each such A defines a bounded operator on $l^2(\Gamma)$ via the usual formula for matrix multiplication:

$$A\xi(s) = \sum_{r\in\Gamma} A(s,r)\xi(r), \quad \text{for } \xi \in l^2(\Gamma).$$

These will be referred to as *finite width operators*. The collection of finite width operators is a $*$-subalgebra of $\mathcal{B}(l^2(\Gamma))$. The *uniform Roe algebra* of Γ, denoted $UC^*(\Gamma)$, is the closure of the $*$-algebra of finite width operators. It is a C^*-algebra The quasi-isometry class of (Γ, d) determines $UC^*(\Gamma)$, which is therefore independent of the choice of generators.

Every $t \in \Gamma$ acts on $l^2(\Gamma)$ by the left regular representation. The action of $t \in \Gamma$ is represented by the matrix A defined by $A(s,r) = 1$ if and only if $s = tr$. Clearly, $t \in \Gamma$ acts as a finite width operator. Thus, $\mathbb{C}[\Gamma] \subseteq UC^*(\Gamma)$, and we have

$$C_r^*(\Gamma) \subseteq UC^*(\Gamma) \subseteq \mathcal{B}(l^2(\Gamma)).$$

Recall that if a unital *-homomorphism, $T : \mathcal{A} \to \mathcal{B}$, between unital C^*-algebras is *nuclear* then there is a net $T_\lambda : \mathcal{A} \to \mathcal{B}$ of finite rank, unital, completely positive linear maps such that $\lim_\lambda \|T_\lambda(x) - T(x)\| = 0$ for all $x \in \mathcal{A}$. It was shown by Kirchberg [15, 9] that a unital C^*-algebra $\mathcal{A}$ is exact if and only if every non-degenerate, faithful representation of $\mathcal{A}$ on a Hilbert space $\mathcal{H}$ provides a nuclear embedding of $\mathcal{A}$ into $B(\mathcal{H})$. In particular, a discrete group Γ is exact if and only if the inclusion of $C_r^*(\Gamma)$ into $\mathcal{B}(l^2(\Gamma))$ given by the left regular representation is a nuclear embedding. The main theorem of this section states that if one restricts the range of the nuclear embedding a little bit, then this strengthened form of exactness implies the uniform embeddability of Γ.

Theorem 3.1. *Let Γ be a finitely generated discrete group. If the inclusion $C_r^*(\Gamma) \subset UC^*(\Gamma)$ is a nuclear map then Γ is uniformly embeddable in a Hilbert space (and hence satisfies the Novikov conjecture).*

Proof. By Theorem 2.3 it is sufficient to produce an approximate unit for $C_0(X \times X; \Delta)$ consisting of positive definite functions. This will be obtained using nuclearity of the inclusion.

There is a general procedure to associate to a linear map $T : C_r^*(\Gamma) \to \mathcal{B}(l^2(\Gamma))$ a function $u : \Gamma \times \Gamma \to \mathbb{C}$ given by the formula

$$u(s,t) = \langle T(st^{-1})\delta_t, \delta_s\rangle,$$

where δ_t denotes the characteristic function of the element $t \in \Gamma$. Note that if T is bounded then $u \in l^\infty(\Gamma \times \Gamma)$. The correspondence

$$\{T : C_r^*(\Gamma) \to \mathcal{B}(l^2(\Gamma))\} \longmapsto \{u : \Gamma \times \Gamma \to \mathbb{C}\}$$

has the following properties:

(i) if T is unital and completely positive then u is positive definite, and
(ii) if $T : C_r^*(\Gamma) \to UC^*(\Gamma)$ has finite rank then $u \in C_0(\Gamma \times \Gamma; \Delta)$.

Further if $T_\lambda : C_r^*(\Gamma) \to \mathcal{B}(l^2(\Gamma))$ is a sequence of bounded linear maps with associated functions u_λ then

(iii) if $\|T_\lambda(x) - x\| \to 0$, for all $x \in C_r^*(\Gamma)$, then $u_\lambda \to 1$ uniformly on $B_R(\Delta)$ for all R.

We verify these properties below, but for now observe that together they imply the desired result. Assuming nuclearity of the inclusion of $C_r^*(\Gamma)$ into $UC^*(\Gamma)$ we obtain unital completely positive maps $T_\lambda : C_r^*(\Gamma) \to UC^*(\Gamma)$ as above. It follows immediately from the properties above that the associated functions $u_\lambda \in l^\infty(\Gamma \times \Gamma)$ form the desired approximate unit.

We now turn to the verification of (i)–(iii), beginning with (i). Let $s_1, \dots, s_n \in \Gamma$ and $z_1, \dots, z_n \in \mathbb{C}$. Define an element of $\mathcal{H} = \oplus l^2(\Gamma)$ by $\xi = (z_1\delta_{s_1}, \dots, z_n\delta_{s_n})$ and an operator on $\oplus l^2(\Gamma)$ by the $n \times n$ matrix $A = [A_{ij}] \in M_n(\mathcal{B}(l^2(\Gamma)))$ where

$$A_{ij} = T(s_i s_j^{-1}).$$

A direct calculation shows

$$\sum_{i,j} \overline{z}_i u(s_i, s_j) z_j = \sum_{i,j} \overline{z}_i \langle T(s_i s_j^{-1})\delta_{s_j}, \delta_{s_i}\rangle = \langle A\xi, \xi\rangle_{\mathcal{H}}.$$

Thus, it suffices to show that A is positive operator on $\mathcal{H}$. However, since T is completely positive, this will follow from the fact that the $n \times n$ matrix $B = [B_{ij}] \in M_n(\mathcal{B}(l^2(\Gamma)))$, where $B_{ij} = s_i s_j^{-1}$ defines a positive operator on $\mathcal{H}$. This is equivalent to the assertion that

$$\sum_{ij} \langle (s_i s_j^{-1}) f_j, f_i \rangle = \|(s_1^{-1} f_1, \ldots, s_n^{-1} f_n)\|^2 \geq 0$$

for all $f_1, \ldots, f_n \in l^2(\Gamma)$, which is straightforward.

We now prove (ii). Since T has finite rank there exist finitely many $f_i \in C_r^*(\Gamma)^*$ and $S_i \in UC^*(\Gamma)$ such that $T = \sum f_i S_i$. Since u depends linearly on T it is sufficient to consider the rank one case where $T(s) = f(s)S$. In this case,

$$|u(s,t)| = |\langle T(st^{-1})\delta_t, /delta_s \rangle| = |f(st^{-1})||\langle S\delta_t, /delta_s\rangle| \leq \|f\|_{C_r^*(\Gamma)^*} |\langle S\delta_t, \delta_s\rangle|,$$

and it suffices show that for all $\varepsilon > 0$ there exists $R > 0$ such that $|\langle \delta_s, S\delta_t \rangle| < \varepsilon$ provided $d(s,t) \geq R$.

At this point the requirement that $S \in UC^*(\Gamma)$ is needed. Let S' be a finite width operator such that $\|S - S'\| < \varepsilon$. Then we have

$$|\langle S\delta_t, \delta_s \rangle| \leq \|S - S'\| + |\langle S'\delta_t, \delta_s\rangle|,$$

and for large enough R, $d(s,t) > R$ forces the last term to be zero. The result follows.

We conclude the proof by verifying (iii). Consider

$$\begin{aligned} u(s,t) - 1 &= \langle T(st^{-1})\delta_t, \delta_s \rangle - \langle \delta_s, \delta_s \rangle \\ &= \langle T(st^{-1})\delta_t - \delta_s, \delta_s \rangle \\ &= \langle (T(st^{-1}) - st^{-1})\delta_t, \delta_s \rangle. \end{aligned}$$

Thus, if we have a family T_λ, it follows that

$$|u_\lambda(s,t) - 1| \leq \|T_\lambda(st^{-1}) - st^{-1}\|.$$

To verify the uniform convergence on sets of the form $B_R(\Delta)$ note that $d(s,t) < R$ implies that st^{-1} lies in a bounded subset of Γ, hence only a finite number of such products are possible. Thus, by taking λ sufficiently large the right side can be made as small as necessary. This completes the proof. □

4. Approximate units and the Haagerup property

The results of Section 2 can be used to directly relate the existence of an approximate unit of positive definite functions to the Haagerup property for the transformation groupoid $\beta\Gamma \rtimes \Gamma$. Here $\beta\Gamma$ is the Stone-Cech compactification of Γ and Γ acts on $\beta\Gamma$ on the right by extending right translation.

This requires extending the notion of positive definite and negative type functions to groupoids. This has been done by Tu [13, Section 3.3] in the following way. We specialize to the case of a transformation groupoid $X \rtimes \Gamma$, defined as above, where X is a compact space on which Γ acts on the right. A complex-valued function φ on $X \rtimes \Gamma$ is *positive definite* if

$$\sum_{ij} \overline{z}_i \varphi(x \cdot s_i, s_i^{-1} s_j) z_j \geq 0, \quad \text{for all } x \in X,\ s_1, \ldots, s_n \in \Gamma \text{ and } z_i, \ldots, z_n \in \mathbb{C}.$$

A real-valued function ψ on $X \rtimes \Gamma$ is of *negative type* if

(i) $\psi(x, e) = 0$ for all $x \in X$,
(ii) $\psi(x \cdot s, s^{-1} t) = \psi(x \cdot t, t^{-1} s)$, for all $x \in X$, s, $t \in \Gamma$,
(iii) $\sum_{ij} a_i \psi(x \cdot s_i, s_i^{-1} s_j) a_j \leq 0$, for all $x \in X$, $s_1, \ldots, s_n \in \Gamma$ and $a_1, \ldots, a_n \in \mathbb{R}$ satisfying $\sum_j a_j = 0$.

Definition 4.1. *The transformation groupoid $X \rtimes \Gamma$ has the* Haagerup property *if there exists a proper, negative type function $\psi : X \rtimes \Gamma \to \mathbb{R}$.*

If $X \rtimes \Gamma$ has the Haagerup property, then it admits a proper affine action on a field of Hilbert spaces, [13]. This latter property, in the case of groups, is call *a-T-menability.* We may now state the main result of this section.

Theorem 4.1. *Let Γ be a finitely generated discrete group. The following are equivalent:*

(i) Γ is uniformly embeddable in a Hilbert space.

(ii) The groupoid $\beta\Gamma \rtimes \Gamma$ has the Haagerup property.

(iii) $C_0(\beta\Gamma \rtimes \Gamma)$ admits an approximate unit of positive definite functions.

Proof. There is an equivalence of groupoids, $\alpha : \Gamma \rtimes \Gamma \to \Gamma \times \Gamma$ given by $\alpha(s,t) = (s, st)$. Here $\Gamma \times \Gamma$ is the trivial groupoid. The inverse of α is $\beta(s,t) = (s, s^{-1}t)$. These maps define correspondences, $\alpha^* : l^\infty(\Gamma \times \Gamma) \leftrightarrow C_b(\beta\Gamma \rtimes \Gamma) : \beta^*$ between functions on $\Gamma \times \Gamma$ and $\beta\Gamma \rtimes \Gamma$ via $\alpha^*(f)(s,t) = f(s, st)$ and $\beta^*(g)(s,t) = g(s, s^{-1}t)$. Note that $\alpha^*(f)$, initially defined on $\Gamma \times \Gamma \subset \beta\Gamma \rtimes \Gamma$, extends by continuity to $\beta\Gamma \rtimes \Gamma$ since $f\alpha(\cdot, t)$ is bounded for each fixed $t \in \Gamma$.

It is easy to check that α^* and β^* are inverses and provide a bijection between $l^\infty(\Gamma \times \Gamma)$ and $C_b(\beta\Gamma \rtimes \Gamma)$. These maps have the following properties which are direct consequences of the definitions.

(i) A function $f \in l^\infty(\Gamma \times \Gamma)$ is metrically proper if and only if $\alpha^*(f)$ is a proper function on $\beta\Gamma \rtimes \Gamma$.

(ii) The map α^* takes the ideal $C_0(\Gamma \times \Gamma; \Delta)$ to the ideal $C_0(\beta\Gamma \rtimes \Gamma)$.

(iii) A net $\{u_\lambda\}$ is an approximate unit for $C_0(\Gamma \times \Gamma; \Delta)$ if and only if $\alpha^*(f)(u_\lambda)$ is an approximate unit for $C_0(\beta\Gamma \rtimes \Gamma)$.

It remains to note that α^* preserves positive definite and negative type functions. This also follows in a straightforward way from the above formulas.

Now the result follows from Theorem 2.3. $\square$

5. Remarks

A finitely generated discrete group Γ is *strongly exact* if the inclusion of $C_r^*(\Gamma)$ into $UC^*(\Gamma)$ given by the left regular representation is a nuclear map, that is, if Γ satisfies the hypothesis of Theorem 3.1.

1. If a discrete group Γ has the property that there is a nuclear embedding of $C_r^*(\Gamma)$ into $B(l^2(\Gamma))$ then the inclusion given by the left regular representation is also nuclear. It is possible that this inclusion is also a nuclear map into $UC^*(\Gamma)$. In other words, it is possible that every exact group is strongly exact. If this is

indeed the case then one would deduce that an exact group satisfies the Novikov conjecture.

2. One may consider algebras A satisfying

$$C_r^*(\Gamma) \subseteq A \subseteq UC^*(\Gamma)$$

and impose the requirement that the inclusion of $C_r^*(\Gamma)$ into A be a nuclear map; if $A = UC^*(\Gamma)$ then Γ is strongly exact, whereas if $A = C_r^*(\Gamma)$ then $C_r^*(\Gamma)$ is nuclear. One obtains a family of conditions interpolating between strong exactness and nuclearity. In the case $A = C_r^*(\Gamma)$ the procedure employed above for constructing a proper negative type function on $\Gamma \times \Gamma$ actually yields an invariant one, which descends to Γ showing that Γ has the Haagerup property. This gives an alternate account of the result of Beki, Cherix and Valette [3].

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Department of Mathematical Sciences, IUPUI, Indianapolis, IN 46202-3216
E-mail address: eguentner@math.iupui.edu

Department of Mathematical Sciences, IUPUI, Indianapolis, IN 46202-3216
E-mail address: kaminker@math.iupui.edu