ERRATA IN A PRIMER OF SUBQUASIVARIETY LATTICES

Here are the errors we have found so far in *A Primer of Subquasivariety Lattices.*

(1) Page 99, line -4 should say "Thus Pre($\mathbf{G}$) *contains* the two preclops in that figure." In fact, there is a third preclop not drawn in Figure 4.3, with $\gamma(s) = v$ and $\gamma(x) = 1$, so that $\mu < \gamma < \lambda$.

(2) Page 99, line -2 would be better as "In Example A.4 of Sect. 4.1 ..."

(3) Pages 167–169. The assignment of predicates and finding the laws of $\mathcal{K}$ are described here. For each compact element $p \in S$ except $\hat{z} = 0_{\mathbf{S}}$, there is a predicate P in the language corresponding to p. In Step 6 of the construction, the predicate $\mathcal{O}$ (or W) assigned to $1_{\mathbf{S}}$ is interpreted so that $\mathcal{O}(x)$ becomes $x \approx e$ for a constant e. In order for this to make sense, $1_{\mathbf{S}}$ should be compact. This was not made clear in the Primer, though it was implicit in the paper Nation [91] on which this part is based.

(4) Page 193 The hypothesis of (ß) in Lemma 7.19 should include that $\mathbf{S}$ is finitely generated.

(5) Page 194, line 6, $\ker \sigma$ could also contain relations $P(u)$ with $u \in W \setminus S$ that do not hold in $\mathbf{U}$. This does not affect the conclusion.

(6) Page 194. Theorem 7.20, when properly dissected, actually describes sufficient conditions for Step 6 of a shortstyle representation to work. This omission has been supplied in a later paper of Hyndman and Nation. It can be stated thusly:

Theorem 1. *Assume* $\mathbf{L} \cong \mathrm{S_p}(\mathbf{S}, H)$ *with* $1_{\mathbf{S}}$ *compact and satisfying one of the following properties. Then there exists a quasivariety* $\mathcal{K}$ *in a language with equality such that* $\mathbf{L} \cong \mathrm{L_q}(\mathcal{K})$*.*

(2) *H consists of a single operator h satisfying* $h^2(x) = h(x) \geq x$.

(3) *The operators of H form a right-zero semigroup, i.e.,* $hk(x) = k(x)$ *for all h, k and at least one $h \in H$ is increasing,* $h(x) \geq x$.

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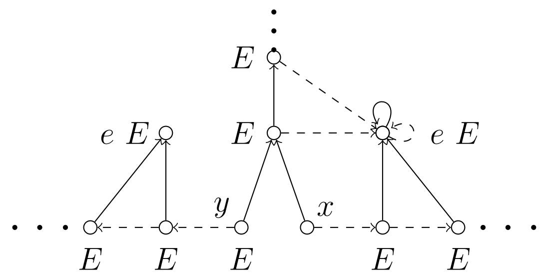

FIGURE 1. A 2-generated quasicritical structure $\mathbf{W}$ in the quasivariety $\mathcal{N}$ of Example 7.31. To simplify the drawing, the constant e appears twice. The predicate E holds at every element except x.

(4) *The operators are increasing, $h_j(x) \geq x$, and form a finite chain under composition, $h_1 < h_2 < \cdots < h_k$ so that $h_i h_j = h_{\max(i,j)}$.*

(7) Page 195, it would be useful to add that the structure in Figure 7.23 satisfies $\eta^3 x = \eta^2 x$.

(8) Page 196, Figure 7.24 should also include $\hat{a} \,\&\, \hat{b} \to \hat{z}$ and $\hat{a} \,\&\, \hat{c} \to \hat{z}$.

(9) Page 206, Figure 7.35 the dotted loop on $\kappa(x)$ should be a solid loop ($\mathcal{K}$ satisfies $\kappa^2(x) = \kappa(x)$).

(10) There are some problems in Section 7.6, which consists of 4 examples of so-called *mediumstyle* representations.

Example 7.29 is correct as it stands.

The statement of Example 7.30 is correct. The claim in the proof that "k-generated structures in $\mathcal{M}$ consist of $m \leq k$ components ... glued over $\{e\}$" is incorrect. More complicated gluings can occur in $\mathcal{M}$, but Lemma 7.19 with the condition (ß)′ applies to all of them, and in fact every quasicritical structure in $\mathcal{M}$ is 1-generated.

Example 7.31 is wrong. This example attempts to represent the lattice $\mathbf{L}_{15} = (\mathbf{3} \times \mathbf{3})[m]$ as $\mathrm{L_q}(\mathcal{K})$ for a quasivariety with equality. The quasivariety $\mathcal{N}$ presented there contains a 2-generated quasicritical structure $\mathbf{W}$, given in Figure 1. In the notation of the example (see Figure 7.46), $\mathbf{W}$ is a subdirect product of $\mathbf{S}$, $\mathbf{T}$ and $\mathbf{U}$, so it is in $\mathcal{N}$. However, $\mathbf{U} \not\leq \mathbf{W}$ making $\mathbf{W}$ quasicritical.

Example 7.32 depends on Example 7.31, so it is also in doubt. We still do not know whether $\mathbf{L}_{15}$ or the pair $(\mathbf{W}, \mu)$ can be represented as subquasivariety lattices.

(11) Page 227, line -3, should read: if $H \leq K$, then $\mathrm{S_p}(\mathbf{S}, K) \leq \mathrm{S_p}(\mathbf{S}, H)$.
(12) Page 262, lines 12–13 claim that the operator γ_4 on $\mathbf{N}$ does not have a longstyle representation. A slight modification of the subsemilattice representation given there does in fact yield a longstyle representation of $(\mathbf{N}, \gamma_4)$. This will be included in a paper of Hyndman and Nation.
(13) Page 263. $\mathbf{B}_3[c]$ can be represented as $\mathrm{Sub}(\mathbf{S}, \wedge, 1, H)$, but the construction given in Figure A.2 is wrong. This is rectified in the upcoming paper of Hyndman and Nation.