Due Friday November 8.

1. Let $H \leq G$.
   (a) The normalizer of $H$ is $N(H) = \{x \in G : xHx^{-1} = H\}$. Prove that the normalizer is a subgroup of $G$ containing $H$.
   (b) Show that $xHx^{-1} = yHy^{-1}$ iff $xN(H) = yN(H)$.
   (c) So the number of conjugates of $H$ in $G$ is ..... 
   (d) Let $H < G$ with $G$ a finite group. Show that $G$ is not the union of $H$ and its conjugates. (Clark)

2. Show that a group $G$ is abelian if and only if the map $\varphi(g) = g^{-1}$ is an automorphism. Show that $G$ is abelian if and only if the map $\psi(g) = g^2$ is an endomorphism. (Clark)

3. Prove the following commutator identities.
   (a) $[xy, z] = [x, z]^y[y, z]$.
   (b) $[x^y, [y, z]][y^z, [z, x]][z^x, [x, y]] = 1$.
   Here $[x, y]$ denotes the commutator and $w^z = z^{-1}wz$.

4. Prove that there are no simple groups of order 42 or order 56.

5. Let $G$ be an infinite simple group and let $H$ be a proper subgroup of $G$. Show that the index of $H$ in $G$ is infinite. (Do not re-invent the wheel.)

6. Show that no group of order $p^2q$, with $p$ and $q$ odd primes, is simple.

7. Find all groups of order 35, 37, 38, 39.

8. Let $i$ denote the identity map on $\mathbb{Z}_2$, and let $s$ denote the permutation switching 0 and 1. Let $A_0 = \langle \mathbb{Z}_2, +, i \rangle$ and $A_1 = \langle \mathbb{Z}_2, +, s \rangle$. Prove that $A_0 \times A_1 \cong A_1 \times A_1$, but $A_0 \not\cong A_1$. (ALVIN)